

How To Rearrange an Equation

Sometimes an equation is given in one form and you would like to rearrange it into another form. For example, the conversion formula to convert Celsius to Fahrenheit degrees is: $F = \frac{9}{5}C + 32$. However, I have a whole bunch of Fahrenheit temperatures that I want to convert to Celsius, so I want to rearrange this equation so that “C” is by itself on the left hand side.

My strategy for this is to “unwrap” the equation. Think of this little puzzle: I have some money in bills that I **roll** up, I **place it in a matchbox** and **close the door**. I take a long piece of string and start **wrapping it** around the matchbox until I have a big ball of string. I then hand it to you and ask you, “How much money is there in here?”. How would you solve it? Unwrap the string? Where would you start? At the end of the string? Let’s make a table where we take the words in red above and write their opposites in blue beside them.

Order that I did things	How the puzzle was made	Order to Solve puzzle	The Opposites
1	Roll up the money	4	unroll the money (and count it)
2	place it in a matchbox	3	take it out of the matchbox
3	close the door	2	open the door
4	wrap the string	1	unwrap the string
	Finish		Start

Notice that I did things in the opposite order using the opposites in order to solve this problem.

Now let’s see how the same idea can be used on the Fahrenheit problem above: First of all let’s remember the Order of Operations (sometimes called BEDMAS). Brackets and Bars are done first, Exponents next, the Division and Multiplication from left to right, and finally Addition and Subtraction from left to right.

Order that I did things	How the puzzle was made	Order to Solve puzzle	The Opposites	Show working for each step.
1	Start with C	4	End with C	$C = \frac{5(F - 32)}{9}$
2	Multiply it by 9	3	Divide by 9	$\frac{5(F - 32)}{9}$
3	Divide by 5	2	Multiply by 5	$5(F - 32)$
4	Add 32	1	Subtract 32	$F - 32$
	Finish with F		Start with F	

The problem becomes a bit harder when the variables appear more than once. For instance: $3r + 7t = 5(r - 6)$ and you want to solve it for the variable “r” (in red). First of all, lets get all the terms that contain that variable on the same side. Subtracting $3r$ from both sides gives us: $7t = 5(rt - 6) - 3r$. Now clear all brackets. $7t = 5rt - 30 - 3r$. Now move every term that does not contain an “r” to the other side: $7t + 30 = 5rt - 3r$. Now use factoring to get the variable to appear only once: $7t + 30 = r(5t - 3)$.

Finally, divide both sides by the bracket: $r = \frac{7t + 30}{5t - 3}$

The “magic” of this is in the factoring, for this is usually where most students “freeze”.