

## Using Factoring to Find Cubes Mentally

Earlier in the year I showed how the factoring formula:  $a^2 - b^2 = (a + b)(a - b)$  could be used to do a mental math trick to square anything (April 18, 2009). Today we extend this idea to see how to cube anything.

Let's look at the factoring formula:  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ .

If we take the  $b^3$  to the other side we get:  $a^3 = (a - b)(a^2 + ab + b^2) + b^3$ . Where "a" stands for "any number".

Here is how you can use this to cube a number in your head. Let's say we are to work out:  $13^3$ . Then  $a = 13$ , and I will let  $b = 3$ . Observe the table below:

Col (1)	Col (2)	Col (3)	Col (4)	Col (5)	Col (6)	Col (7)
Question	a	b	a - b	$a^2 + ab + b^2$	Col (4) x Col (6)	Add $b^3$
$13^3$	13	3	$13 - 3 = 10$	$13^2 + 13 \times 3 + 3^2 = 169 + 39 + 9 = 217$	$10 \times 217 = 2170$	$2170 + 3^3 = 2170 + 27 = 2197$

This originally looks laborious, but with practice and small numbers, I was able to pick it up fairly quickly. The fact that everything in column (5) has addition signs helps as well. Below are some more examples:

Col (1)	Col (2)	Col (3)	Col (4)	Col (5)	Col (6)	Col (7)
Question	a	b	a - b	$a^2 + ab + b^2$	Col (4) x Col (6)	Add $b^3$
$13^3$	13	3	$13 - 3 = 10$	$13^2 + 13 \times 3 + 3^2 = 169 + 39 + 9 = 217$	$10 \times 217 = 2170$	$2170 + 3^3 = 2170 + 27 = 2197$
$24^3$	24	4	$24 - 4 = 20$	$24^2 + 24 \times 4 + 4^2 = 576 + 96 + 16 = 688$	$20 \times 688 = 13760$	$13760 + 4^3 = 13760 + 64 = 13824$
$75^3$	75	5	$75 - 5 = 70$	$75^2 + 75 \times 5 + 5^2 = 5625 + 375 + 25 = 6025$	$70 \times 6025 = 421750$	$421750 + 5^3 = 421750 + 125 = 421875$
$16^3$	16	6	$16 - 6 = 10$	$16^2 + 16 \times 6 + 6^2 = 256 + 96 + 36 = 388$	$10 \times 388 = 3880$	$3880 + 6^3 = 3880 + 216 = 4096$
$115^3$	115	5	$115 - 5 = 110$	$115^2 + 115 \times 5 + 5^2 = 13225 + 575 + 25 = 13825$	$110 \times 13825 = 1520750$	$1520750 + 5^3 = 1520750 + 125 = 1520875$