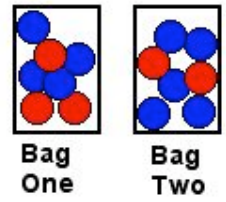
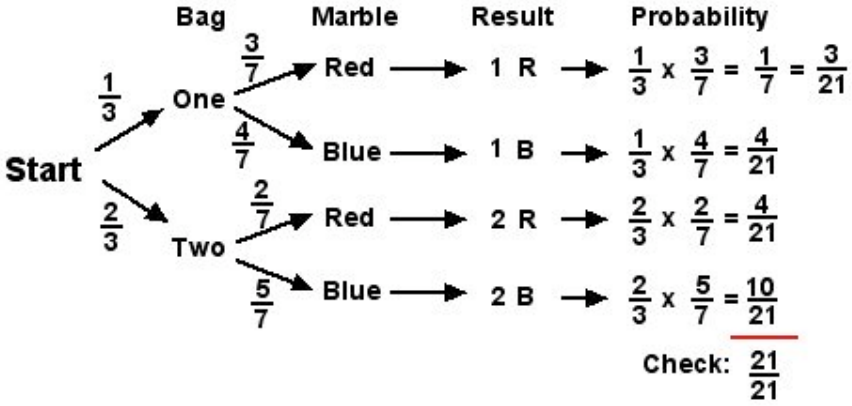
Using a Probability Table to Understand Conditional Probability

A Probability table is a table that looks at all the combinations of a problem PLUS gives you the probabilities of each combination. For example, let's say that bag one contains 3 red marbles and 4 blue marbles, and bag 2 contains 2 red and 5 blue marbles. Roll a single die, if you get 1 or 2, pick from bag one, if you get 3, 4, 5, or 6 you pick from bag two. What is the probability of getting a red marble?

Here is a probability table that diagrams out the problem.





To solve it you add the two fractions that have a "Red" result: $\frac{3}{21}+\frac{4}{21}=\frac{7}{21}=\frac{1}{3}$

If you add up the fractions in the probability column, you must get 1 since the probability of all things happening = 1. We have $\frac{3}{21}+\frac{4}{21}+\frac{4}{21}+\frac{10}{21}=\frac{21}{21}=\frac{1}{1}=1$, so our probabilities are correct.

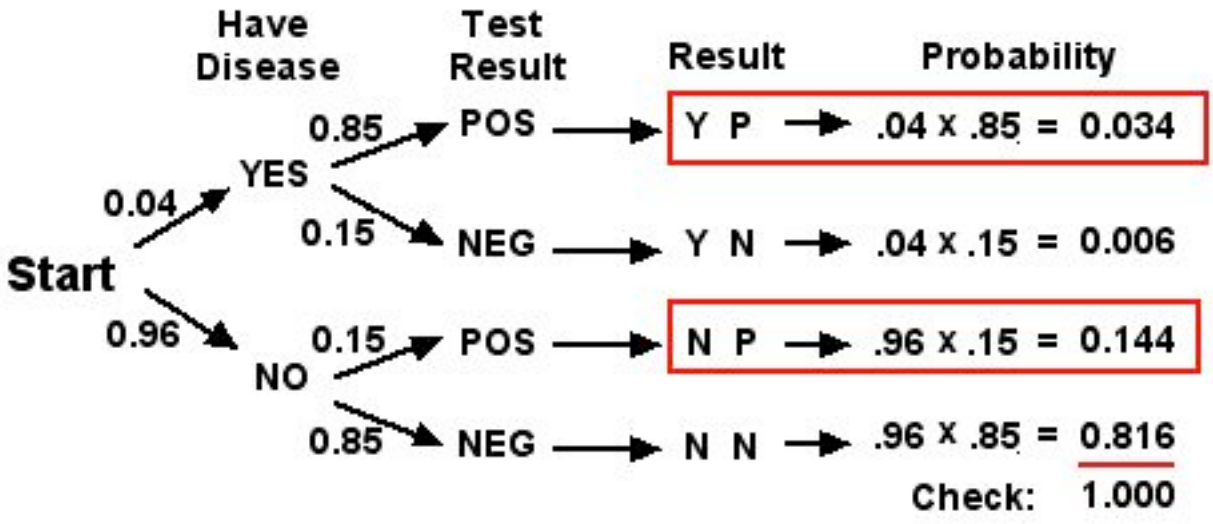
Now let's look at what is called "conditional probability". These problems are quite often worked out with Bayes' Formula, but I will show you how to get the correct answer by looking at a probability table.

Let us say that you have just had some tests done at the hospital to see whether you have the dreaded "math disease". Now, this disease is fatal 90% of the time within 2 years and 100% of the time within 5 years. Therefore, you are very interested in the results. Let us say that the tests themselves are accurate 90% of the time. This means, that if you have "math disease", then 90% of the time the test should be positive (yes – you have it). 10% of the time the test will give you a "False positive", that is, it will say that you have the disease when in fact you DO NOT have it.

If you DO NOT have "math disease", then 90% of the time the test should be negative (no – you DO NOT have it). 10% of the time the test will give you a "False negative", that is, it will say that you DO NOT have the disease when in fact you have it.

Now, the question is, "If you have a positive test result (one that says 'Yes you have the disease'), what is the probability that you actually have it? I will rephrase the question below:

What is the probability that **I have the disease** given that **the test result is a positive** ?
On the next page, you will see a probability table that diagrams out the problem.



To solve this problem, take the words in red after the words "given that" and find that probability first. This probability will become the denominator in your answer. I have put a square red box around the two results that have a "POS" in the Test Result Column. Adding these two probabilities I get: 0.034 + 0.144 = 0.178. This will be my denominator in the final answer.

Now I take the words in blue before the words "given that", and I check ONLY IN THE RED BOXES to find which of those results have a "YES" in the "Have Disease Column". I obtain the probability of 0. 034 and this becomes the numerator of my answer.

Therefore, the answer is $\frac{0.034}{0.034+0.144} = \frac{0.034}{0.178} = 0.19101$, **or** 19.1%

One of the decimals in the denominator will always appear in the numerator.

My suggestion to you, is do NOT go overboard, because of the accuracy of the testing equipment, there still is only a 19.1% chance that you DO have the disease. Get a second opinion, run more tests. (Study harder….).

Conditional probability is probability that occurs when you know something else about the result. I always solve these problems using a probability table (maybe because I am a visual learner). Others will use Bayes' Formula and get the answer that way.

Bayes' Formula is : $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$, **where** $P(B) \neq 0$

This is read as: "The Probability of A occurring given that B has occurred is the probability of both A and B occurring over the probability of B occurring."

I will leave it up to the reader to see which method they find easier.