Multiplying Radicals

The formulas involved with multiplying radicals are as follows:

(1) $\sqrt{n} \times \sqrt{m} = \sqrt{n \times m}$, and the same formula in reverse:

(2) $\sqrt{n \times m} = \sqrt{n} \times \sqrt{m}$

The first one says, "if I am multiplying two roots, then I can multiply the numbers first, and the take the root of the result".

So, $\sqrt{2} = 1.414213562...$, and the $\sqrt{8} = 2.828427125...$ Both of these involve decimals with infinite number of decimal places. Now let's multiply them:

$\sqrt{2} \times \sqrt{8} = 1.414213562 \times 2.828427125$, **or**

$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$

So, multiply a whole bunch of decimals times a whole bunch of decimals and out pops a whole number "4" !

Now remember, the $\sqrt{2}$ doesn't really exist. It has an infinite number of decimals, but you never really find it: there is no number that multiplied by itself equals 2. Even 1.414213562 x 1.414213562 = 1.9999999989447.., it never really reaches 2. However, we do know something about $\sqrt{2}$, we know that if you multiply it by itself, you get "2". Now there are two ways that you could write this:

$\sqrt{2}$ x $\sqrt{2} = 2$. Or $\left(\sqrt{2}\right)^2 = 2$. These lead to two more formulas involved with multiplying decimals:

(3) $\sqrt{a} \times \sqrt{a} = a$ in words: "the square root of anything times itself is the 'anything' ".

(4) $\left(\sqrt{a}\right)^2 = a$ in words: "the square root of anything, squared, is the 'anything' ".

Now, we can look at some multiplying radicals examples:

(1) $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

(2) $3\sqrt{5} \times 2\sqrt{7} = 3 \times 2\sqrt{5 \times 7} = 6\sqrt{35}$. Think, in words, "number times number, roots times roots"

(3) $\sqrt{2} \times \left(\sqrt{5} + 3\sqrt{6}\right) = \sqrt{2 \times 5} + 3\sqrt{2 \times 6} = \sqrt{10} + 3\sqrt{12} = \sqrt{10} + 3\sqrt{4}\sqrt{3} = \sqrt{10} + 6\sqrt{3}$. Notice, on this one that once you get the $\sqrt{12}$, it should be simplified to its lowest terms so $3\sqrt{12} = 6\sqrt{3}$

(4) $\left(4 + \sqrt{3}\right)\left(5 - \sqrt{3}\right) = 4 \times 5 - 4\sqrt{3} + 5\sqrt{3} - \sqrt{3}\sqrt{3} = 20 + 1\sqrt{3} - 3 = 17 + \sqrt{3}$

Here we have the expansion of the two binomials (some may call this FOIL), and then I simplify as far as possible.

(5) $\left(2\sqrt{5} + 3\sqrt{7}\right)^2 = \left(2\sqrt{5} + 3\sqrt{7}\right)\left(2\sqrt{5} + 3\sqrt{7}\right) = 4\sqrt{25} + 6\sqrt{35} + 6\sqrt{35} + 9\sqrt{49}$

$4 \times 5 + 12\sqrt{35} + 9 \times 7 = 20 + 63 + 12\sqrt{35} = 83 + 12\sqrt{35}$

(6) $\left(7+\sqrt{3}\right)\left(7-\sqrt{3}\right) = 7 \times 7 - 7\sqrt{3} + 7\sqrt{3} - \sqrt{3}\sqrt{3} = 49 - 3 = 46$

In this example, we introduce you to the concept of , “the conjugate”. The conjugate is the same numbers in the brackets, but opposite signs. Whenever you multiply these, the two middle terms cancel each other out, and you are just left with the square of the first term MINUS the square of the last term.

Here is a formula that shows this: $(a + b)(a – b) = a^2 – b^2$.

You can see that the two middle terms are the only terms that contain radicals, so all radicals disappear from the answer. We will use this next week, when we look at dividing radicals.