

Dividing Radicals

The two formulas involved with dividing radicals are as follows:

(1) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ which means that instead of taking the square roots first and then dividing, you divide first and take the square root of your answer.

(2) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ which tells you, that to take the square root of a fraction, take the square root of the numerator (the top) and take the square root of the denominator (the bottom).

Here are some examples of these formulas in action:

$$(1) \frac{\sqrt{32}}{\sqrt{8}} = \sqrt{\frac{32}{8}} = \sqrt{4} = 2$$

$$(2) \sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

$$(3) \frac{\sqrt{45}}{\sqrt{9}} = \sqrt{\frac{45}{9}} = \sqrt{5} = 2.236067977 \dots \text{ If you want an "exact" answer we leave it in radical form as: } \sqrt{5}$$

Now, if the fraction does not simplify to a whole number, then we have something like this: $\frac{\sqrt{30}}{\sqrt{7}} = ?$

What we do not want here is to leave the denominator as a radical, or irrational, number. Textbooks will call the following technique. "rationalizing the denominator", which really means, "make the bottom a whole number". To do this, we multiply the top and the bottom by the radical that appears in the denominator. Observe:

$$\frac{\sqrt{30}}{\sqrt{7}} = \frac{\sqrt{30}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{30 \times 7}}{\sqrt{7 \times 7}} = \frac{\sqrt{210}}{\sqrt{49}} = \frac{\sqrt{210}}{7}. \text{ This could be simplified to: } \frac{\sqrt{30}}{\sqrt{7}} = \frac{\sqrt{30}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{210}}{7}$$

We now have a denominator that is a whole number (also a rational number).

Finally, if we have a binomial in the denominator, then we have a problem like this: $\frac{\sqrt{7}}{5 + 2\sqrt{7}} =$

Here we must multiply the top and the bottom by the conjugate of the denominator:

$$\frac{\sqrt{7}}{5 + 2\sqrt{7}} = \frac{\sqrt{7}}{(5 + 2\sqrt{7})} \times \frac{(5 - 2\sqrt{7})}{(5 - 2\sqrt{7})} = \frac{5\sqrt{7} - 2\sqrt{49}}{25 - 10\sqrt{7} + 10\sqrt{7} - 4\sqrt{49}} =$$

$$\frac{5\sqrt{7} - 2 \times 7}{25 - 4 \times 7} = \frac{5\sqrt{7} - 14}{-3}$$

This now leaves us with a rational number in the denominator, because the middle terms cancel.