Adding and Subtracting Radicals

If you have downloaded the poster "10 Big Picture Ideas" from our download section, the number one "Big Picture Idea" is that you can only add and subtract things that are the same. Observe, the following:

(1) 3 **apples** plus 2 **apples** equals 5 **apples** — **things in red are "the same"**
(2) 3**a** + 2**a** = 5**a** — **things in red are "the same"**
(3) 3 **apples** plus 2 **bananas** cannot be simplified — cannot be added, they are not the same
(4) 3**a** + 2**b** = cannot be simplified — cannot be added, they are not the same
(5) 3 **apples** plus 2 **bananas** = 3 **fruit** + 2 **fruit** = 5 **fruit** — **"fruit" is a name they have in common**
(6) 3 **sevenths** plus 2 **sevenths** = 5 **sevenths** — **things in maroon are the same**
(7) $\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$ — fractions, same denominator, can be added

think of denominator as being "de name of"
"denominare" is medieval latin for "to name"

(8) $\frac{3}{7}+\frac{2}{5}=\frac{3\times5}{7\times5}+\frac{2\times7}{5\times7}=\frac{15+14}{35}=\frac{29}{35}$ — fractions with different denominators must first be changed to a "common denominare" or a common denominator in order to be added or subtracted.

Thus, as you can see from the above 8 examples, you can only add (or subtract) things that are the same. If they are not the same, then you try to convert them to something that is the same (**fruit** or 35ths) in order to add or subtract them.

Now, let's see how this applies to adding and subtracting radicals. Remember, when you say something like $\sqrt{2}$, it refers to a certain number, that if you multiply it by itself, you get 2. So,

(1) $3\sqrt{2}+2\sqrt{2}=5\sqrt{2}$ — the $\sqrt{2}$'s are the same, so they can be added
(2) $7\sqrt{2}-3\sqrt{2}=4\sqrt{2}$ — the $\sqrt{2}$'s are the same, so they can be subtracted
(3) $5\sqrt{2}+3\sqrt{7}$ = cannot be simplified — $\sqrt{2}$ and $\sqrt{7}$ are not the same, so they cannot be added
(4) $3\sqrt{2}-2\sqrt{7}$ = cannot be simplified — $\sqrt{2}$ and $\sqrt{7}$ are not the same, they cannot be subtracted

Now finally if we have something like

$5\sqrt{2}+6\sqrt{3}-2\sqrt{2}+8\sqrt{3}=$

$5\sqrt{2}-2\sqrt{2}+6\sqrt{3}+8\sqrt{3}=$

$3\sqrt{2}+14\sqrt{3}$

Here, we have rearranged the order so that the $\sqrt{2}$'s are together and can be subtracted, since they are the same, and the $\sqrt{3}$'s are together and can be added since they are the same. The final line of $3\sqrt{2}+14\sqrt{3}$ cannot be simplified, because they are different.

If we put the idea of simplifying radicals from last week (Nov 4) with today's idea, then we can simplify something like this: (see next page)

$$3\sqrt{24} - 2\sqrt{50} + \sqrt{54} - 4\sqrt{98} =$$
$$3\sqrt{4} \times \sqrt{6} - 2\sqrt{25} \times \sqrt{2} + \sqrt{9} \times \sqrt{6} - 4\sqrt{49} \times \sqrt{2} =$$
$$3 \times 2 \times \sqrt{6} - 2 \times 5 \times \sqrt{2} + 3 \times \sqrt{6} - 4 \times 7 \times \sqrt{2} =$$
$$6\sqrt{6} - 10\sqrt{2} + 3\sqrt{6} - 28\sqrt{2} = 9\sqrt{6} - 38\sqrt{2}$$

Each radical is simplified first and then the $\sqrt{6}$'s are added together to make $9\sqrt{6}$ and the $\sqrt{2}$'s are subtracted to make $-38\sqrt{2}$.

So the general idea is to simplify your radicals as far as possible, then add or subtract those radicals that are the same. Remember, if you have something like this:

$3\sqrt{2} - \sqrt{2}$, then this is the same as $3\sqrt{2} - \mathbf{1}\sqrt{2}$, so our answer would be $2\sqrt{2}$.