

Working With Radicals – Simplifying

Remember, when we are writing a radical, it represents a “picture” that we carry in our mind:

$$\sqrt{16} = \sqrt[4]{16} = (4 \times 4 = 16) = 4$$

$$\sqrt[3]{125} = (5 \times 5 \times 5 = 125) = 5$$

$$\sqrt[4]{16} = (2 \times 2 \times 2 \times 2 = 16) = 2$$

$$\sqrt[5]{243} = (3 \times 3 \times 3 \times 3 \times 3 = 243) = 3$$

So the root index (the little number in the “crook” of the radical sign) tells you how many times identical factors are multiplied out. The brackets above represent what goes on in your mind, however, you could write them out each time.

Now, in order to simplify radicals we are going to look at the first formula for radicals:

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}, \text{ or, in reverse :}$$

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

We will show you two ways to simplify radicals, the first method uses the above formula (line two):

$$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4 \times \sqrt{2} = 4\sqrt{2}$$

$$\sqrt{125} = \sqrt{25 \times 5} = \sqrt{25} \times \sqrt{5} = 5 \times \sqrt{5} = 5\sqrt{5}$$

$$\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3 \times \sqrt{3} = 3\sqrt{3}$$

$$\sqrt[3]{40} = \sqrt[3]{8 \times 5} = \sqrt[3]{8} \times \sqrt[3]{5} = 2 \times \sqrt[3]{5} = 2\sqrt[3]{5}$$

The strategy on these is two factor the number into a perfect root times some other number. The perfect root should be the largest perfect root that is a factor of your original number. Look at the three ways the following simplification is done: We start with an entire radical and end up with a mixed radical.

$$\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$$

$$\sqrt{72} = \sqrt{9} \times \sqrt{8} = 3\sqrt{8} = 3\sqrt{4} \times \sqrt{2} = 3 \times 2\sqrt{2} = 6\sqrt{2}$$

$$\sqrt{72} = \sqrt{4} \times \sqrt{18} = 2\sqrt{18} = 2\sqrt{9} \times \sqrt{2} = 2 \times 3\sqrt{2} = 6\sqrt{2}$$

As you can see, the first line gets us to the simplified answer quicker because although 36, 9 and 4 are all factors of 72, we choose the largest square root to get to our answer quicker. All three give you the same final answer, but the first line, above, is much quicker.

Now the second method is to use a factor tree on the original number as we have done below:

$$\sqrt{72} = \sqrt{2 \times 2 \times 2 \times 3 \times 3}$$

Now think of the radical sign as a “prison”, and the factors as “prisoners”. A prisoner can “escape” and get outside of the prison if he has an equal buddy to help him. However, only one of the equal buddies gets released (the other, I guess, is shot for helping him or her escape). Thus we find that there are two 2’s and two 3’s, so one of them escapes and the remaining “2” has no buddy so he or she does not get out. We now have:

$$\sqrt{72} = 2 \times 3\sqrt{2} = 6\sqrt{2} \text{ Once radicals are “outside” of prison, they multiply, so we get 2 times 3 not 2 plus 3.}$$

This method is especially useful when big numbers are involved. Look at the following problem:

Simplify $3\sqrt{72} \times 6\sqrt{48} = 3 \times 6 \times \sqrt{72 \times 48} = 18\sqrt{3,456}$

Now we have to break down the number 3 456 into a perfect square (preferably the largest perfect square) times some other number. This could take some time. Now look at this problem with the “get out of jail free” method:

Simplify $3\sqrt{72} \times 6\sqrt{48} = 3 \times 6 \times \sqrt{72 \times 48} =$

$$18\sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 3} =$$

$$18 \times 2 \times 2 \times 2 \times 3\sqrt{2 \times 3} = 432\sqrt{6}$$

With this method, I never have to figure out that 576 is the largest perfect square (the square root of 576 is 24) that is a factor of 3 456.