Sequences and Series Part One

The Arithmetic Sequence

An arithmetic sequence is one where each term is found by **adding** or **subtracting** the SAME number each time. Here are some examples:

1, 4, 7, 10, 13, 16, 19, 22, …

10, 8, 6, 4, 2, 0, -2, -4, …

-10, -5, 0, 5, 10, 15, …

1, 3.5, 6, 8.5, 11, 13.5, …

The first term is sometimes labeled "**a**" but is really $\mathbf{t_1}$ (read as "term one").
The difference between each term is labeled "**d**", (this can be found by subtracting, d = any term subtract the term before it). So, in the first example the difference, d = 3, and can be found by 4 – 1, or 7 – 4, or 22 – 19, etc.
Then number of terms is labeled "**n**", and the "nth" term is labeled $\mathbf{t_n}$.

The four variables above are linked with your first formula: $\mathbf{t_n = t_1 + (n - 1)\, d}$.

I'll use it first with an example and then we will analyze it. Let's take the first sequence above and ask the question, "What is the 52nd term in the sequence?".

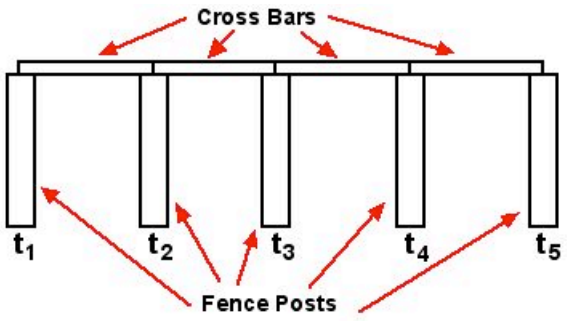
(1) Start by filling the variables with their values:
$\mathbf{t_1 = 1,\ d = 3,\ n = 52,\ t_n = t_{52} = ?????}$

Now fill in the formula: $\mathbf{t_{52} = 1 + (52 - 1)\ 3}$ or
$t_{52} = 1 + 51 \times 3$, or $t_{52} = 1 + 153$ or $\mathbf{t_{52} = 154}$.

Now, think of the formula as a series of fence posts as illustrated in the diagram to the right:

So, if you look at the formula above in red, and the diagram, you can sort of see how it all fits together.

Now, we have four variables, and any one of them can be the unknown. I illustrate this with the three examples on the next page.

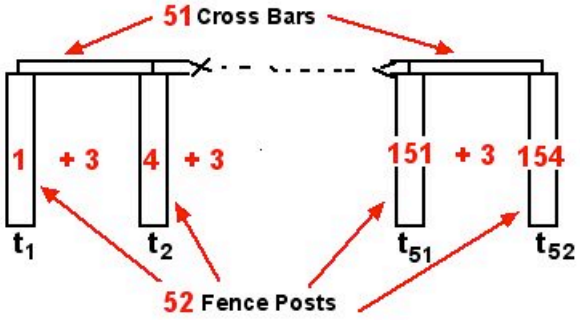


Start with term one (the first fence post) and go 4 cross bars to get to 5th fence post (term 5)

Note, the number of cross bars is one less than the number of fence posts. Think of cross bars as being the difference between each term, then you can read this formula as

term 5 = term 1 + (number of terms - 1) times difference

So if we want the 52 term, (52nd fence post) we will have 51 cross bars, or 51 x 3 = 153 after our starting value of 1. Hence the 52nd term is 154.



(2) If the 73rd term is 1 845, and the first term is 43, what is the difference between each term?

Start by filling the variables with their values: $\mathbf{t_1 = 43, d = ???, n = 73, t_n = t_{73} = 1\ 845}$

Now fill in the formula: $\mathbf{1845 = 43 + (73 - 1)\ d}$ or 1845 – 43 = 72 d or 1802 = 72 d and dividing both sides by 72 we get: $\mathbf{d = 1802 / 72 = 25.03}$. So the difference is 25.03.

(3) If I have a difference of – 5, and a first term of 68 and a last term of – 122, how many terms are there?
Start by filling the variables with their values: $\mathbf{t_1 = 68, d = -5, n = ???, \text{and } t_n = -122}$

Now fill in the formula: $\mathbf{-122 = 68 + (n - 1)\ ^{-}5}$ or - 122 - 68 = -5(n-1) or - 190 = -5(n-1) and dividing both sides by - 5 we get: $\mathbf{d = -190 / {}^{-}5 = 38}$. So there are 38 terms.

(4) If the 7th term is 54 and the 16th term is 135, then what is the first term?

We are looking at it like this ___, ___, ___, ___, ___, ___, **54**, ___, ___, … ___, **135**,
t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_{15} t_{16}
t_1 t_2 t_9 t_{10}

I first of all start by sliding the term numbers to the right so that 54 becomes term one. These are in red.
Start by filling the variables with their red values: $\mathbf{t_1 = 54, d = ???, n = 10, \ t_n = t_{10} = 135}$

Now fill in the formula using the values as delimitated in red.
$\mathbf{135 = 54 + (10 - 1)d}$ or 135 – 54 = 9d or 81 = 9d and dividing both sides by 9 we get: **d = 9.**
So now we can subtract 9 from 54 until we get to term 1 or we can say (using the black terms above)
$\mathbf{t_1 = ??, d = 9, n = 7, \ t_n = t_7 = 54}$
Now fill in the formula using the values as delimitated in black.
$\mathbf{54 = t_1 + (7 - 1)9}$, **thus we get** $\mathbf{54 = t_1 + 6 \times 9}$ **and** $\mathbf{54 = t_1 + 54}$, **or** $\mathbf{54 - 54 = t_1}$, **or** $\mathbf{0 = t_1}$

Thus, the first term would be equal to zero.

Now another question that I have seen is a "rephrasing" of the question we have just done. It looks like this: Find the eight arithmetic means between 54 and 135. Here is the diagram:

54, ___, ___, ___, ___, ___, ___, ___, ___, **135**
t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10}

So working like we did above, we get d = 9, so now add 9 onto 54 and keep adding 9. The arithmetic means are now in red: 54, 63, 72, 81, 90, 99, 108, 117, 126, 135
t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10}

You can check your answer by adding 9 onto your last answer of 126, to make sure that you do get to 135.