Probabilities of Lotteries

The probabilities and odds of winning at lotteries can be easily worked out using a key on almost all scientific calculators. The key is labeled $_nC_r$ and is called the Combination key. It is used for when you are picking "r" objects from a pool of "n" objects, where the order that they are picked does NOT count. Most lotteries fall into this. If the order DOES count, then the $_nP_r$ key is used, where the P = Permutations.

We will use today the Lotto 6-49 from Canada, where you are to pick 6 numbers out of a pool of 49. The total number of combinations is therefore $_{49}C_6$. The actual formula $_nC_r$ for is below.

$$_nC_r = \frac{n!}{(n-r)!r!} = \frac{49!}{(49-6)!6!} = \frac{49 \times 48 \times 47 \times 46 \times 45 \times 44 \times 43!}{43! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 13{,}983{,}816$$, where the 43!'s cancel.

If you use your calculator your steps are 49 $_nC_r$ 6 = 13 983 816. Each calculator has a different way to get to the $_nC_r$ function. On the TI-83 and TI-84, you press MATH, use arrow keys to get to PROB and choose 3 to get the $_nC_r$ function.

Now, the neat use of these functions are if you are asking the question, what is the probability of getting 3 of my 6 numbers chosen, or 5 of my 6 chosen? Those questions can be answered easily be thinking of two pools, one are your special numbers (let's say you chose 1, 2, 3, 4, 5, 6) and those numbers that are NOT your special numbers (the 43 numbers from: 7, 8, 9, 10,…, 47, 48, 49). The formula then becomes:

$_6C_n$ x $_{43}C_{49-n}$ where n = number of your numbers that you are interested in. Here are all the possibilities:

How many of your numbers	How many that are NOT your numbers	Total number of Combinations	Total # of all combinations	Probability	Odds, 1 chance in:
6	0	$_6C_6$ x $_{43}C_0$ = 1	**13 983 816**	7.15×10^{-8}	13 983 816
5	1	$_6C_5$ x $_{43}C_1$ = 258	**13 983 816**	1.85×10^{-5}	54 201
4	2	$_6C_4$ x $_{43}C_2$ = 13 545	**13 983 816**	9.69×10^{-4}	1 032.4
3	3	$_6C_3$ x $_{43}C_3$ = 246 820	**13 983 816**	0.01765	56.7
2	4	$_6C_2$ x $_{43}C_4$ = 1 851 150	**13 983 816**	0.132378	7.6
1	5	$_6C_1$ x $_{43}C_5$ = 5 775 588	**13 983 816**	0.413019	2.42
0	6	$_6C_0$ x $_{43}C_6$ = 6 096 454	**13 983 816**	0.435965	2.29
Total		**13 983 816**		1.0	1

So, since in the Lotto 6-49 you start to win with 3 out of your 6 numbers chosen (or 2 + bonus which is not shown above), the chances that you don't win anything is close to 0.9813 or a 0. 0186 probability that you might win at least something. Including the bonuses (seen below) you have a 1 in 32.3 chance of winning something. (1 + 258 + 13 545 + 246 820 + 6 + 172 200 = 432 830, and 13 983 816 ÷ 432 830 = 32.307)
If you include the bonus, then you have 7 chosen numbers by the lottery, (6 numbers plus 1 bonus).
So, the probability of having 5 of your numbers, plus the bonus number, and none from the other 42 numbers is:

$_6C_5$ x $_1C_1$ x $_{42}C_0$ = 6 and probability = 6/13983816 = 4.291×10^{-7} or 1 chance in 2 330 636

So, the probability of having 2 of your numbers, plus the bonus number, and 3 from the other 42 numbers is:

$_6C_2$ x $_1C_1$ x $_{42}C_3$ = 172 200 and probability = 0.012314 or 1 chance in 81.2

By the way, I have checked my numbers against the Lotto 6/49 website and we agree accept for 5 out of 6, where they say the odds are 1 chance in 55 492, whereas I have 1 chance in 54 201. They are correct in

that the chances that you have 5 numbers from your chosen 6 and NOT the bonus number and your other number coming from the remaining 42 comes into play. The odds of this happening are now:

$_6C_5$ x $_1C_0$ x $_{42}C_1$ = 252 and probability = 252/13983816 = 1.802 x 10^{-5} or 1 chance in 55 491.3

So, of the 258 combinations listed above in the table for 5 out of 6, 6 of them also contain the bonus. Subtract those 6 from 258 and you get the 252 I have shown in the working above.

So a new table would look like this:

How many of your numbers	NOT your numbers	Total number of Combinations	Total # of all combinations	Probability	Odds, 1 chance in:
6	**0**	**$_6C_6$ x $_{43}C_0$ = 1**	**13 983 816**	7.15 x10^{-8}	13 983 816
5 + bonus	**1**	**$_6C_5$ x $_1C_1$ x $_{42}C_0$ = 6**	**13 983 816**	4.3 x 10^{-7}	2 330 636
5 (no bonus)	**1**	**$_6C_5$ x $_1C_0$ x $_{42}C_1$ = 252**	**13 983 816**	1.85 x10^{-5}	55 491.3
4	**2**	**$_6C_4$ x $_{43}C_2$ = 13 545**	**13 983 816**	9.69 x10^{-4}	1 032.4
3	**3**	**$_6C_3$ x $_{43}C_3$ = 246 820**	**13 983 816**	0.01765	56.7
2 + bonus	**4**	**$_6C_2$ x $_1C_1$ x $_{42}C_3$ = 172 200**	**13 983 816**	0.012314	81.2
2 (no bonus)	**4**	**$_6C_2$ x $_1C_0$ x $_{42}C_4$ = 1 678 950**	**13 983 816**	0.120064	8.33
1	**5**	**$_6C_1$ x $_{43}C_5$ = 5 775 588**	**13 983 816**	0.413019	2.42
0	**6**	**$_6C_0$ x $_{43}C_6$ = 6 096 454**	**13 983 816**	0.435965	2.29
Total		**13 983 816**		1.0	1