Some Number Puzzles

Here are a couple of weird results, why do they happen?

(1) Powers of 11:

$$11^0 = 1$$
$$11^1 = 1\ 1$$
$$11^2 = 1\ 2\ 1$$
$$11^3 = 1\ 3\ 3\ 1$$
$$11^4 = 1\ 4\ 6\ 4\ 1$$
$$11^5 = 1\ 6\ 10\ 5\ 1$$
$$11^6 = 1\ 7\ 7\ 15\ 6\ 1$$

Pascal's Triangle:
(Each number is sum of numbers above left and right of it)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The two are almost the same (or are the the same?), but why?

(2) Weird reversals:

Take any 3 digit numbers where the 3 digits are in descending order. Reverse the digits and **SUBTRACT** your two results. Your answer will always be a multiple of 99, why? Now reverse the digits of your answer and **ADD** it to your answer, and you always end up with 1 089, why?

Answers to last week's questions

(1) The following problem came from Mike Williams, a 10^{th} grade math student from Salinas, California. It was printed in the December, 1961 issue of Recreational Mathematics Magazine, p. 60.

Mike observed that: "The area between two concentric circles is the same as that of a trapezoid which has the respective circumferences as their parallel bases and an altitude that is the difference between the radii." Can you prove that?

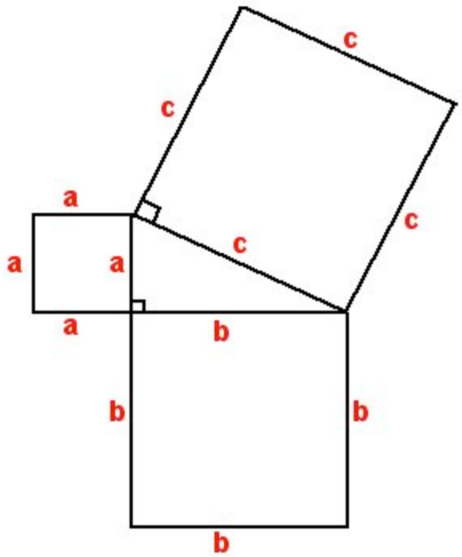
Proof: Using "C and R" and "c and r" as circumferences and radii of the big circle and the little circle respectively. The area of the trapezoid would be : $A_T = \frac{(C+c)(R-r)}{2}$ and the area between the two concentric circles is: $A_C = \pi R^2 - \pi r^2$, or $\pi(R^2 - r^2)$, or $\pi(R-r)(R+r)$.

If they are equal then $A_C = A_T$ or: $\pi(R-r)(R+r) = \frac{(C+c)(R-r)}{2}$. Now divide both sides by (R – r) and you get: $\pi(R+r) = \frac{(C+c)}{2}$.

Multiply both sides by 2 and simplifying gets you: $2\pi(R+r) = (C+c)$, **or** $2\pi R + 2\pi r = C + c$.

This obviously works out since C = 2πR and c = 2πr, the circumferences of the respective circles.

(2) As you know, from Pythagoras, if "a", "b" and "c" are sides of a right triangle, where "c" is the hypotenuse and "a" and "b" are the sides, then $c^2 = a^2 + b^2$. You can draw this as I have below with the actual squares being drawn on each side. Think of the formula as: $\mathbf{c}^{\text{squared}} = \mathbf{a}^{\text{squared}} + \mathbf{b}^{\text{squared}}$



Now, does the same thing apply if you drew equilateral triangles on each side or semi-circles on each side. Are the areas still the same? The "new Pythagoras" formulas would be: $\mathbf{c}^{\text{equilateraled}} = \mathbf{a}^{\text{equilateraled}} + \mathbf{b}^{\text{equilateraled}}$ **and** $\mathbf{c}^{\text{semi-circled}} = \mathbf{a}^{\text{semi-circled}} + \mathbf{b}^{\text{semi-circled}}$.

<u>Proof</u>: $\mathbf{c}^{\text{equilateraled}} = \mathbf{a}^{\text{equilateraled}} + \mathbf{b}^{\text{equilateraled}}$, see the next page:

The area of an equilateral triangle is: $A_{equil} = \frac{\sqrt{3}x^2}{4}$, where x = the side of the equilateral triangle.

Therefore: $\mathbf{c}^{\text{equilateraled}} = \mathbf{a}^{\text{equilateraled}} + \mathbf{b}^{\text{equilateraled}}$ becomes:

$\frac{\sqrt{3}c^2}{4} = \frac{\sqrt{3}a^2}{4} + \frac{\sqrt{3}b^2}{4}$, **so multiply each side by** $\frac{4}{\sqrt{3}}$, **gives you:**

$c^2 = a^2 + b^2$, **which of course, we know is true.**

<u>Proof:</u> of $\mathbf{c}^{\text{semi-circled}} = \mathbf{a}^{\text{semi-circled}} + \mathbf{b}^{\text{semi-circled}}$.

The Area of a Semi-circle is $A_{semi} = \frac{\pi R^2}{2}$, so the formulas for each sides would be:

$$\frac{\pi\left(\frac{c}{2}\right)^2}{2} = \frac{\pi\left(\frac{a}{2}\right)^2}{2} + \frac{\pi\left(\frac{b}{2}\right)^2}{2}$$, **or** $$\frac{\pi\left(\frac{c^2}{4}\right)}{2} = \frac{\pi\left(\frac{a^2}{4}\right)}{2} + \frac{\pi\left(\frac{b^2}{4}\right)}{2}$$, **or**

$\pi\left(\frac{c^2}{8}\right) = \pi\left(\frac{a^2}{8}\right) + \pi\left(\frac{b^2}{8}\right)$, **now multiply both sides by** $\frac{8}{\pi}$**:**

$c^2 = a^2 + b^2$, **which we know is true**

(3) Here are a couple more Alphametics. Remember each different letter stands for a different digit from 0 to 9. No number begins with a "0", and no letter can have two different values. When the letters are replaced by the proper digit, each question is a "true" addition problem.

MOON	9552
MEN	902
CAN	382
REACH	10836

ALLS	9332	OR	9332
WELL	8433		8433
THAT	6596		6096
ENDS	4072		4572
SWELL	28433		28433