# <u>Using Canceling Units to Solve Problems</u>

A lot of times, when you look at a problem, you may not know how to proceed. Do I divide by this, or multiply by that? The answer, quite often, is in the units. Here is an example problem that illustrates this:

I went into a car dealership to purchase a new car. The advertising card on the car said that it got 5.6 L/100 Km, which means 5.6 Litres per 100 kilometres. Now, I am of a bit older generation, so I wanted to know how many miles per gallon I was going to get. The salesperson could not answer that right away. After I left (I did end up buying the car though), I sat down to work out a conversion formula for them. Here is the work:

Remember, the word "per" means "divided by", so 5.6 L/100 km means: $\frac{5.6\ \mathbf{L}}{100\ \mathbf{km}}$, **or** $\frac{100\ \mathbf{km}}{5.6\ \mathbf{L}}$.

My final answer needed to be in Miles per Gallon, which means: $\frac{\mathbf{Mi}}{\mathbf{G}}$.

I also had the following conversion tables:

For changing Canadian and British Gallons to Litres: $\frac{4.55\ \mathbf{L}}{1\ \mathbf{gal}}$, **or** $\frac{1\ \mathbf{gal}}{4.55\ \mathbf{L}}$

For changing United States Gallons to Litres: $\frac{3.79\ \mathbf{L}}{1\ \mathbf{gal}}$, **or** $\frac{1\ \mathbf{gal}}{3.79\ \mathbf{L}}$ since 1 US gal is 0.83 British gal (or 6 US = 5 British)

For changing between Miles and Kilometres we use: $\frac{1.609\ \mathbf{km}}{1\ \mathbf{mi}}$, **or** $\frac{1\ \mathbf{mi}}{1.609\ \mathbf{km}}$

For each conversion, I have also put it as a reciprocal, so I can choose either one. Now I need to convert so that I end up with $\frac{\mathbf{Mi}}{\mathbf{G}}$, that is, the Miles on the numerator and the Gallons on the denominator. So I start with:

$\frac{1\ \mathbf{mi}}{1.609\ \mathbf{km}}$ as that puts the "Miles" on the numerator. Now I multiply this by: $\frac{100\ \mathbf{km}}{5.6\ \mathbf{L}}$ to get:

$\frac{1\ \mathbf{mi}}{1.609\ \mathbf{km}} \times \frac{100\ \mathbf{km}}{5.6\ \mathbf{L}}$. In this way, the Units "km" on top will cancel the units "km" on the bottom. Now I am going to multiply by: $\frac{4.55\ \mathbf{L}}{1\ \mathbf{gal}}$ to get:

$\frac{1\ \mathbf{mi}}{1.609\ \mathbf{km}} \times \frac{100\ \mathbf{km}}{5.6\ \mathbf{L}} \times \frac{4.55\ \mathbf{L}}{1\ \mathbf{gal}}$, so now the "L =Litres" unit on top cancels the "L = Litres" unit on the bottom.

Simplifying this multiplication of fractions we get:

$\frac{1 \times 100 \times 4.55\ \mathbf{mi}}{1.609 \times 5.6\ \mathbf{gal}} = \frac{50.4972\ \mathbf{mi}}{1\ \mathbf{gal}}$, **or about 50.5 mi/gal** (This is the British Gallon.)

Now, to find a general "formula" to change any value from litres/100 kilometres to miles/gallon, just change the 5.6 to a "1" and we get the "magic" number:

$$\frac{1\ \mathbf{mi}}{1.609\ \mathbf{km}} \times \frac{100\ \mathbf{km}}{1\ \mathbf{L}} \times \frac{4.55\ \mathbf{L}}{1\ \mathbf{gal}} = \frac{1 \times 100 \times 4.55\ \mathbf{mi}}{1.609 \times 1 \times 1\ \mathbf{gal}} = \frac{282.784\ \mathbf{mi}}{1\ \mathbf{gal}}.$$

So, I can now convert using 282.784 by doing the following:

Divide 282.784 by the number of litres per 100 km to get miles per gallon. So, 282.784 ÷ 5.6 = 50.4972, the same answer we got above. So, a closer approximation is the remember the magic number 283. If I divided 283 by 5.6, I get 50.5, which compares favourably to 50.4972.

The reverse uses the same number. If I had 40 miles per gallon, I divide 283 by 40 and I get 7.08 Litres per 100 kilometres, or 7.08 L / 100 Km.

So, in Canada, or Britain, the magic number is 282.784 or **283**.

Doing the conversion for the United States, I get:

$$\frac{1\ \mathbf{mi}}{1.609\ \mathbf{km}} \times \frac{100\ \mathbf{km}}{1\ \mathbf{L}} \times \frac{3.79\ \mathbf{L}}{1\ \mathbf{gal}} = \frac{1 \times 100 \times 3.79\ \mathbf{mi}}{1.609 \times 1 \times 1\ \mathbf{gal}} = \frac{235.55\ \mathbf{mi}}{1\ \mathbf{gal}}.$$

Thus the magic number in the US is 235.55 or **236**.

So, if I divide 236 by 5.6, I get 42.1 miles per US gallon. So 5.6 L per 100 km equals 42.1 miles per US gallon.

I first learned this “cancel the units” technique in my Grade 11 Chemistry class. (Thanks Mr. Robertson !) and I have used it successfully all the way through University and my teaching career. At University, I majored in Mathematics, but also took courses in Physics, Chemistry and Astronomy, so I found it invaluable.

On Sunday, of this week, we are going to look at a series of problems called the “Fermi” problems. I use this idea a lot, in solving those types of “weird” problems. See you Sunday.