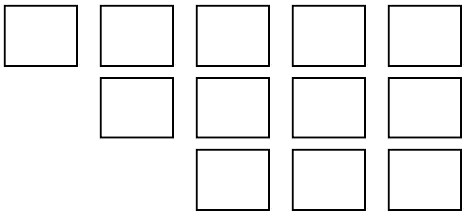
Three New Puzzles For You

(1) Two fathers gave their sons some money. One father gave his son $90.00 and the other father gave his son $80.00. When the two sons counted their wealth they found that together they had become wealthier by only $90.00! What happened to all that money?

(2) The problem in the following game is to determine the strategy which will enable one of the players to force the other to lose. Place 12 squares (or pennies) in the following arrangement of three rows:



Two players move alternately and their turn a player may remove as many squares as desired, provided they are from the same row. The player taking the last square, loses the game. Which player, the first or the second, can guarantee himself or herself a win, and by what strategy?

(3) There are five hats, three red and two black, which are to be placed on the heads of two other persons and you. All three of you know how many hats there are and their colours. Three chairs are placed in a single file, all facing the same direction. You are in the front seat with the other two persons occupying the chairs behind you. All of you are blindfolded and a hat is placed on your heads. The two remaining hats are removed, and when the blindfolds are removed, no one is to turn around and look behind him.

The person in the third chair from the front is asked their colour, but they do not know. The person in the second chair from the front is asked their colour, but they, also, do not know. What colour hat do you have on and why?

Answers to Last Week's Puzzles

(1) Powers of 11:

$$11^0 = 1$$
$$11^1 = 1\ 1$$
$$11^2 = 1\ 2\ 1$$
$$11^3 = 1\ 3\ 3\ 1$$
$$11^4 = 1\ 4\ 6\ 4\ 1$$
$$11^5 = 1\ 6\ 10\ 5\ 1$$
$$11^6 = 1\ 7\ 7\ 15\ 6\ 1$$

Pascal's Triangle:
(Each number is sum of numbers above left and right of it)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The two are almost the same (or are the the same?), but why?

The binomial theorem states that each row of Pascal's Triangle can be found by:

$(a-b)^n = \sum_{r=1}^{n} {}_nC_r a^{n-r} b^r$. Since 11^n can be considered the binomial $(10 + 1)^n$, then we have Pascal's triangle involved.

If n = 0 then you get $11^0 = 1$
If n= 1 then you have $11^1 = (10 + 1)^1 = \mathbf{1} \times 10 + \mathbf{1} = 11$
If n= 2 then you have $11^2 = (10 + 1)^2 = \mathbf{1} \times 10^2 + \mathbf{2} \times 10 + \mathbf{1} = 100 + 20 + 1 = 121$
If n= 3 then you have $11^3 = (10 + 1)^3 = \mathbf{1} \times 10^3 + \mathbf{3} \times 10^2 + \mathbf{3} \times 10 + \mathbf{1} = 1\,000 + 300 + 30 + 1 = 1331$
If n= 4 then you have $11^4 = (10 + 1)^4 = \mathbf{1} \times 10^4 + \mathbf{4} \times 10^3 + \mathbf{6} \times 10^2 + \mathbf{4} \times 10 + \mathbf{1} = 10\,000 + 4\,000 + 600 + 40 + 1 = 14641$
The rest of the rows work the same remembering carry overs. So
$11^5 = (10 + 1)^5 = \mathbf{1} \times 10^5 + \mathbf{5} \times 10^4 + \mathbf{10} \times 10^3 + \mathbf{10} \times 10^2 + \mathbf{5} \times 10 + \mathbf{1} = 100\,000 + 50\,000 + 10\,000 + 1000 + 50 + 1 = = 100\,000 + (50\,000 + 10\,000) + 10 \times 100 + 50 + 1 = \mathbf{1}00\,000 + \mathbf{6}0\,000 + \mathbf{10} \times 100 + \mathbf{5}0 + \mathbf{1}$

The second question:

(2) Weird reversals: Take any 3 digit numbers where the 3 digits are in descending order. Reverse the digits and **SUBTRACT** your two results. Your answer will always be a multiple of 99, why? Now reverse the digits of your answer and **ADD** it to your answer, and you always end up with 1 089, why?

(3)

Here is an example: Take 841, reversing the digits, I get 148. Now subtract 841 – 148 and I get 693. Reverse these digits and I get 396. Now add 693 + 396 and you end up with 1 089!!!

How it Works:

Let's write the original number as 100H + 10T + 1U, where H = hundred's digit, T = ten's digit and U = unit's digit. If I reverse this number, it becomes 100U + 10T + 1H. Subtracting these two results leads us to:

(100H + 10T + 1U) – (100U + 10T + 1H) = 100H + 10T + 1U – 100U – 10T – 1H = 99H – 99U = 99(H – U)

Thus the result will be a multiple of 99 where you multiply 99 by a number from 2 to 8 since H and U are at least 2 apart. Below are those multiples:

99 x 2 = 198	99 x 3 = 297	99 x 4 = 396	99 x 5 = 495
99 x 6 = 594	99 x 7 = 693	99 x 8 = 792	99 x 9 = 891

Now if you look at the results above, all of them have the ten's digit = 9 (so T = 9), and the hundred's plus the unit's digits add up to 9 (so H + U = 9, or H = 9 – U). So, if we add the number HTU to the digits reversed, UTH, then we get the following:

(100H + 10T + 1U) + (100U + 10T + 1H) = 101H + 20T + 101U. Now substitute it T = 9 and H = 9 – U and we get:
101(9 – U) + 20x9 + 101U = 909 – 101U + 180 + 101U = 909 + 180 = 1 089

HaHaHa !! Now matter what we start with (as long as the 3 digits are in descending order) we end up with the same answer, the magical number 1 089 !!