

Using Matrices to do Power Rating in Sports

A matrix is a rectangle of numbers, with a dimension of row x column. Here are some examples:

$$A = \begin{bmatrix} 1 & 3 & 5 & -7 \end{bmatrix} \text{ a } 1 \times 4 \text{ matrix}$$

$$B = \begin{bmatrix} 3 & -2 & 5 \\ 4 & 3 & -2 \end{bmatrix} \text{ a } 2 \times 3 \text{ matrix}$$

$$C = \begin{bmatrix} 1 & 0 & 5 \\ 2 & -3 & 3 \\ 5 & 2 & -5 \end{bmatrix} \text{ a } 3 \times 3 \text{ square matrix}$$

You can add and subtract equal dimensioned matrices by adding or subtracting the corresponding elements of the matrix. So, as shown below: like colours get added:

3	2	-2	0
7	-4	5	3

Let the above matrix be A

5	1	4	-3
2	0	9	1

Let the above matrix be B

5	3	2	-3
9	-4	14	4

Then the above matrix is A + B

You multiply matrices if the columns of the first equals the rows of the second. You take the row of the first, turn it 90 degrees clockwise and multiply by a column of the second. The result will be a matrix that has the rows of the first and the columns of the second. Here is an example.

A =

3	2	-1
2	-3	0

B =

5	1	3	-2
3	1	1	-3
-4	0	4	2

Then A x B =

25	5	7	-14
1	-1	3	5

So we start with Row 1 x Column 1 = $3 \times 5 + 2 \times 3 + -1 \times -4 = 15 + 6 + 4 = 25$. The answer goes into (1, 2) of the answer. Likewise with Row 2 x Column 3 = $2 \times 3 + -3 \times 1 + 0 \times 4 = 6 + -3 + 0 = 3$. The answer goes into (2, 3) of the answer.

So, what does this have to do with sports? Read on to the next page.

If you make up a square matrix for the results of some sports tournament, where 0 = Loss, and 1 = Win, we might have the following:

Lions defeated Braves and Cheetahs, but lost to Dodgers and Hawks.

Braves lost to everyone except the Cheetahs.

Cheetahs beat Dodgers only.

Dodgers defeated the Lions and Braves, but lost to the Hawks.

There were no ties.

Here is the five by five matrix:

Let's call it "A". This is sometimes called a "dominance" matrix.

	Lions	Braves	Cheetahs	Dodgers	Hawks
Lions	0	1	1	0	0
Braves	0	0	1	0	0
Cheetahs	0	0	0	1	0
Dodgers	1	1	0	0	0
Hawks	1	1	1	1	0

Then $A^2 =$

0	0	1	1	0
0	0	0	1	0
1	1	0	0	0
0	1	2	0	0
1	2	2	1	0

This represents when one team beats another team, and that team in turn, has beaten a third team. This is called two-stage dominance. So the matrix A^2 gives you the number of two-stage dominances between the teams. Now lets form another matrix by adding the two matrices above to give us $A^2 + A$. This gives us a power rating of the teams, which we will define as the sum of all the one and two-stage dominances.

So Then $A^2 + A =$

					Sum of each row	Team Name
0	1	2	1	0	4	Lions
0	0	1	1	0	2	Braves
1	1	0	1	0	3	Cheetahs
1	2	2	0	0	5	Dodgers
2	3	3	2	0	10	Hawks

The numbers in blue, now give you the power ratings of each of the five teams.

Now, I did the above matrix multiplication by hand (to remind myself how I had to do it in the past) and by using the matrix keys on my TI -84 Plus Texas Instruments calculator.

You also have matrix multiplication built into Microsoft Excel spreadsheets (and I am sure other spreadsheets as well). On the next page I will show you how we can do the above on MS Excel.

Matrix A	Lions	Braves	Cheetahs	Dodgers	Hawks
Lions	0	1	1	0	0
Braves	0	0	1	0	0
Cheetahs	0	0	0	1	0
Dodgers	1	1	0	0	0
Hawks	1	1	1	1	0

Matrix A

Copied	0	1	1	0	0
Again	0	0	1	0	0
	0	0	0	1	0
	1	1	0	0	0
	1	1	1	1	0

First of all above, I plugged in the values of the matrix A, and then I copied the result by using this formula in cell B9: = B2. Then I filled right the rest of that row and the next four columns by filling down. By using formulas, and a spreadsheet, then I can reuse this with different data and the results will get worked out.

Then I did the A² matrix by going to cell B16 and using the formula: =MMULT(B2:F6,B9:F13)The formula in the example must be entered as an array formula. First, type the formula into cell B16 and then press RETURN. The single result is 0. Next, select the range B16:F20, press CONTROL+U, and then press Command (apple key)+RETURN. The result is below:

A x A

0	0	1	1	0
0	0	0	1	0
1	1	0	0	0
0	1	2	0	0
1	2	2	1	0

Then I added the two last matrices by going to cell B23 and using the formula: = B9 + B16. Using the fill across and fill down functions I got the matrix below:

A x A + A					Sum of Each Row
0	1	2	1	0	4
0	0	1	1	0	2
1	1	0	1	0	3
1	2	2	0	0	5
2	3	3	2	0	10

Finally I went to H23 and typed in the following formula: =SUM(B23:F23). I then filled this down to get the sum of each row. Pulling the results together with the names and then sorting them by going under the “Data” and using the sort command I finally got the following:

Team	Power Ranking	Sorted Results
Lions	4	Hawks 10 First
Braves	2	Dodgers 5 Second
Cheetahs	3	Lions 4 Third
Dodgers	5	Cheetahs 3 Fourth
Hawks	10	Braves 2 Fifth