

Using Boolean Algebra to Solve Problems

When George Boole was working on his algebra, in about 1855, he noticed the similarity in the Addition and Multiplication tables and the Truth Tables of “OR” and “AND”.

Below is an arithmetic table:

Addition	0	1		Multiplication	0	1
0	0	1		0	0	0
1	1	2		1	0	1

Below is a truth table in logic

OR	FALSE	TRUE		AND	FALSE	TRUE
FALSE	FALSE	TRUE		FALSE	FALSE	FALSE
TRUE	TRUE	TRUE		TRUE	FALSE	TRUE

As you can see, these two tables are very similar, in fact if you can say $0 = \text{FALSE}$ and $1 = \text{TRUE}$ and then adjust it slightly so that $1 + 1 = 1$ (and not 2), because you cannot get more “TRUE” than “TRUE”, then the first table becomes the one below and the two tables are virtually identical.

Below is an “adjusted” arithmetic table:

Addition	0	1		Multiplication	0	1
0	0	1		0	0	0
1	1	1		1	0	1

We can use these tables to solve puzzles. The following comes from an article by J. A. H. Hunter called *Some Inferential Problems*, that appeared in the Recreational Mathematics Magazine, February, 1961. I was in grade 8 when my Dad subscribed these magazines to me, and although I only received 6 of them, I still have 4 of them after 48 years! Here is a puzzle with a solution.

Say we have two conflicting statements about the name of a boy, and we know that each statement contains one mistake. One said “Jack Dibble” and the other said “John Dibble”.

Obviously Dibble was his name, and his first name was neither Jack nor John. But let us see how this would be handled by Boolean Algebra.

We only have the two numerical values, 0 and 1. There’s nothing more true than “true”; if in the course of the working, we arrive at any number greater than “one”, we must make it equal to “one”.

Let A stand for Jack, B for John and C for Dibble. Then we can represent each statement in two ways:

Multiplication: IF both A and C were equal to 1 (i.e. true), then the product $AC = 1$. BU if either A or C has a value of 0 (i.e. false), the product $AC = 0$.

Addition: If either A or C (or both) has a value of 1 (i.e. true), then $A + C = 1$.

Now, on the basis of the two statements we can say $(A + C)(B + C) = (1)(1) = 1$ and multiplying out the brackets we get: $AB + AC + BC + C^2 = 1$. But we know each statement contains one mistake, so $AC = 0$ and $BC = 0$. Obviously $AB = 0$, hence we are left with $C^2 = 1$, i.e. $C = 1$, which tells us that the boy’s name is Dibble. Furthermore, as $C = 1$, and $AC = 0$ and $BC = 0$, we see that $A = 0$ and $B = 0$. This confirms that his first name was neither Jack nor John. Now onto the next page for today’s puzzle.

There is an island name Kalota, where the Kalotan women conform rigidly to the strange custom that a woman must never make two consecutive true or untrue statements: if one statement is true, then her next must be a lie, and vice versa.

Now a merchant had four attractive daughters: Kassa, Kessa, Kissa, and Kossa. There are no twins on Kalota, and the girls had this to say about their ages:

Kassa started it: "Kissa is twenty-two, and Kessa twenty-one." Kessa's version was quite different: "Kossa is nineteen," she told me, "and Kissa is twenty-one."

Kissa replied, "Kassa is twenty-one, and Kossa eighteen."

One of the four girls was eighteen, can you figure out their ages?

Answer, using Boolean Algebra, next week!