Gauss – Bolyai – Lobachevski, Who Gets Credit?

I first really studied Geometry in Grade 10. During those years, early 1960's, we took Algebra for grades 9, 11 and 12, but the whole grade 10 year was for Geometry. We probably were exposed to an axiomatic system then, but as a typical Grade 10 student, I didn't really GROK it. (To "GROK" is a word from Robert Heinlein's book, ***Stranger in a Strange Land***. To me, it means, "To rrrreeeaaaalllllyyyy understand". To understand it so well, that it is part of you and will never be forgotten. Like riding a bike.) I carried on, after Grade 10, with algebra, trigonometry and then in University, Calculus and more algebra and trigonometry. Then in my third year of University, September, 1967, at the tender age of 19, I walked into two Math courses that had a profound change on my life. One, was a computer course where I started to learn how to program a computer in a language called FORTRAN. It was the start of a beautiful relationship, as computers have been a big part of my life since then. The other course was on "Projective Geometry". This is the Geometry that is used to find perspective in art, where there are no parallel lines, for what we see as parallel lines do meet, at infinity. Interestingly, both of these courses were taught by the same professor. This professor, Ernie Cockayne, was also a great jazz banjo player and singer, (who appeared at our jazz dance nights at the rugby club), and the topped ranked amateur tennis player in Victoria, BC. I decided then, that here was something worthwhile to emulate.

The concept of Projective Geometry was that if you change an axiom or a postulate, in this case the Euclidean Parallel Postulate, then all of the conclusions that were generated from that postulate change. One of them, was that there were no longer 180 degrees in a triangle. This geometry was one of the many that sprung from the change away from Euclidean Geometry. These geometries were labeled "Non-Euclidean Geometry". What follows is the story of its beginnings.

Johann Karl Friedrich Gauss was a German mathematician (1777 – 1855) that I wrote about in Thursday, May 18, 2009. He had been a mathematical prodigy who has gone down as one of the top three mathematicians who have ever lived. (Archimedes and Newton being the other two). However, if he had one flaw, it was that he was reluctant to publish unless he was absolutely sure of the results. The Parallel Postulate of Geometry was very controversial and much more verbose than the other 9 postulates and axioms. For over 1 700 years, mathematicians had tried to prove it from the other nine. The problem boiled down to the following:

Through a point "P", that is not on line "AB", there is (are):

(1) Only one line drawn through point "P", that is parallel to line AB, or
(2) No lines that can be drawn through point "P", that are parallel to line AB, or
(3) More than one lines drawn through point "P", that are all parallel to line AB.

One way of doing a proof was called "Reductio ad Asurbum", or reduce the logic to an absurdity. To use this method, one assumes the opposite to what you want to prove is true. Then logical deductions are done from that assumption. Eventually, we hope, we get logically to a statement that contradicts something else that we know, or have assumed, is true. This is the absurdity. Therefore, our original assumption must be false, and its exact opposite must be true. Which is great, because this is what we wanted to prove as true all along! A great example of this is Euclid's proof that the $\sqrt{2}$ is irrational. I'll add that later to this post.

Now, if you wanted to prove Euclid's Parallel postulate (which is the first of the choices listed above), then what you can do is assume that #2 is true, and logically make deductions until a contradiction appears. Then assume #3 is true and logically make deductions until a contradiction appears with it as well. Since #2 and #3 cannot be true, then #1 must be true. Voila !! Now many of times people tried these, and usually either abandoned the proof, or "faked" a contradiction. The "fake" was soon spotted by other mathematicians.

Now, what Guass did, was find out that NO contradiction appeared, on either #2 or #3. In other words, all three might be true. If you define a line as "the shortest distance between two points", then you have the following:

(1) If only one parallel exists, then you have classical "Euclidean" Geometry, which works fine on a flat plane. Sometimes people called this "Plane Geometry". The sum of the angles of a triangle is equal to 180°.
(2) If no lines are parallel, then you have Projective Geometry, or the Geometry of a curved surface such as the Earth. The shortest distance between two lines on the surface of the Earth is a great circle, and all great circles meet in two points. An example is the 120 degree West Longitude line which meets all other lines of Longitudes at the North and South Poles. If you start at the North Pole and go South down the 0 degree longitude line through Greenwich, England, until you hit the Equator. Now turn 90° right and head West until you meet the 90° longitude and then turn 90° right again. You are now heading North again, and will meet up at the North Pole with your starting line. You will come into the North Pole 90° away from the starting line. Thus you have formed a triangle that has three, 90° angles in it. The only thing different, you are no longer on a flat plane, but instead you are on a curved surface. The sum of the angles of a triangle is more than 180°.
(3) Here we more have than one, maybe as infinite number of parallel lines. Imagine a surface made up of the two bells of trumpets pressed together. Line AB is on one trumpet, and point "P" is on the other trumpet. As a line approaches the join, it "falls away" and never crosses over the join, and hence never intersects AB, and then it must be parallel. Obviously, there are an infinite number of such lines. The sum of the angles of a triangle is less than 180°.

Now Gauss stumbled upon this, BUT NEVER published his results. Janos Bolyai, was a Hungarian mathematician (1802 – 1860). He was the son of mathematician that knew Gauss, and the younger Bolyai and Gauss became friends. He was reputed to be a good violinist and an excellent duelist. In 1825, he began to look at the proof of the Parallel Postulate, and in 1831, he added an extra 26-page appendix onto his father's book, about his discoveries. Gauss praised the younger Boyai's work, but then told him that he had done the same work some years earlier, but had not published it!

Meanwhile, in Russia, Nikolai Ivanovich Lobachevsky (1793 – 1856) had published, in Russian, his version of Non-Euclidean Geometry. He was lecturing on it as early as 1826. Lobachevsky published his work in 1829, but it was not seen in rest of Europe until after Boyai's work was published.

Thus, the debate begins. Who should get credit, the German, Gauss (whose work was discovered after his death), or the Hungarian, Bolyai, who published first in Europe, or the Russian, Lobachevsky, who actually published first, in Russian, and much later translations appeared in French and German.

I leave it to the readers to argue this one out. I know, that what it did to me, was to understand that if other people had different "core beliefs", they're "axioms or postulates", then their view of the world would be different than mine. AND, this taught me, that BOTH might be true! Pretty cool thing for a 19 year old to realize, and something that has stayed with me, my whole life. Thank you, Ernie Cockayne.