Divisibility by 7 Trick – As Easy as 1-3-2

When you see a divisibility rules in a Math book, they traditionally give divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10. I gave you the divisibility by 11 "trick" or rule on Saturday, March 7, 2009. But, if you look at this list, where is the divisibility by 7 rule? It is usually left out, in that the rule seems more complicated than actually dividing by 7 and seeing whether there is a remainder.

However, I have a slightly easier rule, that with a bit of practice, is quite quick. And, of course, I will show you why it works. Here is the rule – just remember the sequence, from right – to – left, of 1 – 3 – 2. I will illustrate with the two numbers below, one with 6 digits and one with 5 digits. Starting from the right and moving left, I place the numbers 1 and 3 and 2 under each digit of the number.

First number	Second number
382,296	74,089
231 231	31 231

Now, multiply each red digit by the digit above it and add the results. Now multiply the first group on the right by **+ 1** and multiply the second group from the right by **– 1** and add the results.

First number: **- 1**(2 x 3 + 3 x 8 + 1 x 2) **+ 1**(2 x 2 + 3 x 9 + 1 x 6) = **- 1**(32) **+ 1**(37) = **-** 32 **+** 37 = 5

Since 5 is not divisible by 7, the original number (382 296) is not divisible by 7. In fact, 382296 ÷ 7 = 54613.7142857… or 54 613 and remainder of 5. So our answer above, also gives us the remainder left over when we do divide by 7. Remember to be divisible by 7, means that remainder is "0" when you divide by 7.

Second number: **- 1**(3 x 7 + 1 x 4) **+ 1**(2 x 0 + 3 x 8 + 1 x 9) = **- 1**(25) **+ 1**(33) = **-** 25 **+** 33 = 8

Since 8 is not divisible by 7, the original number (74 089) is not divisible by 7. In fact, 74089 ÷ 7 = 10584.142857… or 10 584 and remainder of 1. So our answer above, also gives us the remainder left over when we do divide by 7 (since 8 ÷ 7 leaves a remainder of "1").

If you had a number with more than 9 digits you just keep repeating the right-to-left sequence of 1 – 3 – 2 and alternating the multiplier in front from positive 1 to negative 1, to positive 1, etc.

Why it works:

Let's look at a three-digit number first. We can write a three-digit number, "HTU", as 100H + 10T + 1U, where H = Hundred's digit, and T = Ten's digit and U = Unit's digit. So if we check are dividing by 7, we get:

$$\frac{100\boldsymbol{H}+10\boldsymbol{T}+1\boldsymbol{U}}{7}=\frac{(98+2)\boldsymbol{H}+(7+3)\boldsymbol{T}+1\boldsymbol{U}}{7}=$$

$$\frac{98\boldsymbol{H}+2\boldsymbol{H}+7\boldsymbol{T}+3\boldsymbol{T}+1\boldsymbol{U}}{7}=\frac{98\boldsymbol{H}+7\boldsymbol{T}+2\boldsymbol{H}+3\boldsymbol{T}+1\boldsymbol{U}}{7}=$$

$$\frac{98\boldsymbol{H}+7\boldsymbol{T}}{7}+\frac{2\boldsymbol{H}+3\boldsymbol{T}+1\boldsymbol{U}}{7}=\frac{7(14\boldsymbol{H}+\boldsymbol{T})}{7}+\frac{2\boldsymbol{H}+3\boldsymbol{T}+1\boldsymbol{U}}{7}$$

Now since the first fraction in the above answer, $\frac{7(14\boldsymbol{H}+\boldsymbol{T})}{7}$ is obviously divisible by 7, then if the original number "HTU" is divisible by 7 comes down to whether $\frac{2\boldsymbol{H}+3\boldsymbol{T}+1\boldsymbol{U}}{7}$ has a remainder of "0". Read on:

Now, for larger numbers lets say a 6 digit number “ABCDEF”. In expanded form this would be: 100,000A + 10,000B + 1,000C + 100D + 10E + 1F. So if we check are dividing by 7, we get:

$$\frac{100{,}000A + 10{,}000B + 1{,}000C + 100D + 10D + 1F}{7} =$$

$$\frac{(100{,}002 - 2)A + (10{,}003 - 3)B + (1{,}001 - 1)C + (98 + 2)D + (7 + 3)E + 1F}{7} =$$

$$\frac{100{,}002A - 2A + 10{,}003B - 3B + 1{,}001C - 1C + 98D + 2D + 7E + 3E + 1F}{7} =$$

$$\frac{100{,}002A + 10{,}003B + 1{,}001C + 98D + 7E - 2A - 3B - 1C + 2D + 3E + 1F}{7} =$$

$$\frac{7(14{,}286A + 1429B + 153C + 14D + 1E) - 1(2A + 3B + 1C) + 1(2D + 3E + 1F)}{7} =$$

$$\frac{7(14{,}286A + 1429B + 153C + 14D + 1E)}{7} + \frac{-1(2A + 3B + 1C) + 1(2D + 3E + 1F)}{7}$$

Now the first fraction in our answer, $\frac{7(14{,}286A + 1429B + 153C + 14D + 1E)}{7}$, is obviously divisible by 7, so if the original number is divisible by 7, depends on whether $\frac{-1(2A + 3B + 1C) + 1(2D + 3E + 1F)}{7}$ has a remainder of “0”. So, if you look closely at the numerator of this last fraction you can see, reading from the right, the coefficients have the pattern one – three – two, with the sign in front of the bracket alternating from **+ 1** to **– 1**.

Now, also, the remainder you get from dividing the last term by 7, is going to be the same as the remainder you get when you divide the original number by 7.

This concept can be carried on with more digits, such as “ABCDEFGHI” and so on.

So, with practice, you can check whether a number is divisible by 7, by remembering the following phrase: “It’s as easy as **1 – 3 – 2**” !!!