

Twenty Question Game

Here is a neat thing to do with a group of one to a class of 20 or 30. Tell the person that, “I am thinking of a whole number from 1 to a million (1 000 000), inclusive. That means it could be 1 or 1 000 000 or any whole number in between them. You have 20 questions to get the number, and I can only answer ‘Yes’ or ‘No’ to the question.”

Soon, into this game, they will realize that just guessing a number such as saying, “Is it 254 834?”, is not going to get them anywhere. Instead, they must frame a question such as, “Is it greater than 500 000”, and then the answer of ‘Yes’ or ‘No’ gives them equally as much information. Because $2^{20} = 1\,048\,576$, then:

$$\left(\frac{1}{2}\right)^{20} = \frac{1}{1\,048\,576} \quad \text{and so dividing one million in half 20 times will narrow the search down to only}$$

one number. By the way, you may want to start with a number between 1 and 1 000 and only 10 guesses (since $2^{10} = 1\,024$). After they get this, move to a number between 1 and 1 000 000 and twenty questions and then to really knock their socks off, try a number between 1 and one billion (1 000 000 000) and thirty questions. Since $2^{30} = 1\,073\,741\,824$, they should be able to get it in 30 questions as long **as they don't vary from the strategy.**

Answers to last week's puzzles

- (1) **From Games Magazine: Square Dance:** Sally invited 17 guests to her party. She assigned each guest a number from 2 to 18, and kept the number 1 for herself. When everyone was dancing, Sally noticed that the sum of each couple's numbers was a perfect square. What was the number of Sally's partner?

I started by setting up this table: I chose the perfect squares, and what two number could add to them.

Square →	25	16	9	4
Combination	18 & 7	15 & 1	8 & 1	3 & 1
Combination	17 & 8	14 & 2	7 & 2	
Combination	16 & 9	13 & 3	6 & 3	
Combination	15 & 10	12 & 4	5 & 4	
Combination	14 & 11	11 & 5		
Combination	13 & 12	10 & 6		
Combination		9 & 7		

Here are the numbers: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Firstly, 18, 17 and 16 appear only once, so I have put them in RED above and crossed out the numbers 7, 8, 9, 16, 17 and 18, since they now have their dance partners. I have also crossed out any combinations that 7, 8 or 9 appeared in as they cannot happen any more. I made these blue.

Secondly, I looked at the combination of 3 & 1. If this happens, then 6 & 3 cannot happen, thus 10 & 6 must happen, so 15 & 10 can not happen. That leaves 15 to go with 1, but 3 was with 1, so therefore we have an error. Thus 3 & 1 can not happen, so 15 must be with 1 (I've made this green). So 15 & 10 and 3 & 1 can not happen (made it blue). Therefore 10 must be with 6 (green), and thus 6 cannot be with 3 (blue). This leaves 13 to go with 3 (green) and therefore 13 can not be with 12 (blue). Therefore 12 must be with 4 (green) and thus 5 cannot be with 4 (blue). Now we are almost done, 11 must be with 5, so 14 & 11 can not happen (blue). This leaves 14 to be with 2 and we are all paired up. So Sally's partner is #15.

(2) **From Math Teacher Magazine:** The total number of angles in **two** regular polygons (all sides and all angles are equal) is 13, and the total number of diagonals is 25. How many sides does each polygon have? (see answer on next page).

Answer: Let the number of sides be “a” and “b”, so since the sides and angles of each polygon are equal, then for our first equation we have: (1) $a + b = 13$ or $b = (13 - a)$. The number of diagonals of each polygon is:

Sides \times (sides $- 3$) divided by 2. Thus the diagonals of a polygon with sides of “a” = $\frac{a(a - 3)}{2}$, and the

diagonals of a polygon with sides of “b” = $\frac{b(b - 3)}{2}$. Therefore our second equation is:

$$(2) \frac{a(a - 3)}{2} + \frac{b(b - 3)}{2} = 25 \text{ or } a(a - 3) + b(b - 3) = 50, \text{ or } a^2 - 3a + b^2 - 3b = 50.$$

Substituting in $b = (13 - a)$ we get $a^2 - 3a + (13 - a)^2 - 3(13 - a) = 50$. Squaring out and simplifying we get to the quadratic equation: $a^2 - 13a + 40 = 0$ and $a = 5$ or 8 . Thus $b = 8$ or 5 .

So the answer is we have a pentagon with 5 angles, 5 sides and 5 diagonals; and an octagon with 8 angles, 8 sides and 20 diagonals. Sure enough the sides add up to 13 ($5 + 8 = 13$), and the diagonals add up to 25 ($5 + 20 = 25$).

(3) **Cariboo Math Contest, 1989:** In a generous mood, David gave half the money he happened to have to Jim. Not to be outdone, Jim then gave one quarter of the he **then** had (he already had some of his own) to David. They now each ended up with 75 cents. How much money did they each have to start with?

Answer: Let’s do this in reverse, and think “backwards”. If David and Jim both ended up with 75 cents then it looks like this:

Transaction:	DAVID	JIM
1		
2		
3	75 cents	75 cents

Thus, since Jim had given David one-quarter of his money, he must have had 100 cents ($100 - 25 = 75$). For David to have ended up with 75 cents, he must have had 50 cents at stage 2. So our table looks like this:

Transaction:	DAVID	JIM
1		
2	50 cents	100 cents
3	75 cents	75 cents

And if, in the first transaction, David had given Jim one-half of what he had, then David must have started with 100 cents (100 divide by 2 = 50). If Jim had 100 cents after this first transaction, the he must have had 50 cents to begin with. ($50 + 50 = 100$). Thus the whole series of transactions look like this:

Transaction:	DAVID	JIM
1	100 cents	50 cents
2	50 cents	100 cents
3	75 cents	75 cents

Thus David started with 100 cents and Jim started with 50 cents.

This is an important problem solving lesson, as sometimes the way to solve a problem is to work backwards.