Using Similar Triangles to Find Distances and Heights

I first saw some of these methods at a Boy Scout's camp I attended when I was about 14 or 15. When I started to take Geometry (which, when I went to school, was the whole of my grade 10 year of mathematics), I then began to realize why the techniques worked. I have used these concepts in a variety of different situations since then.

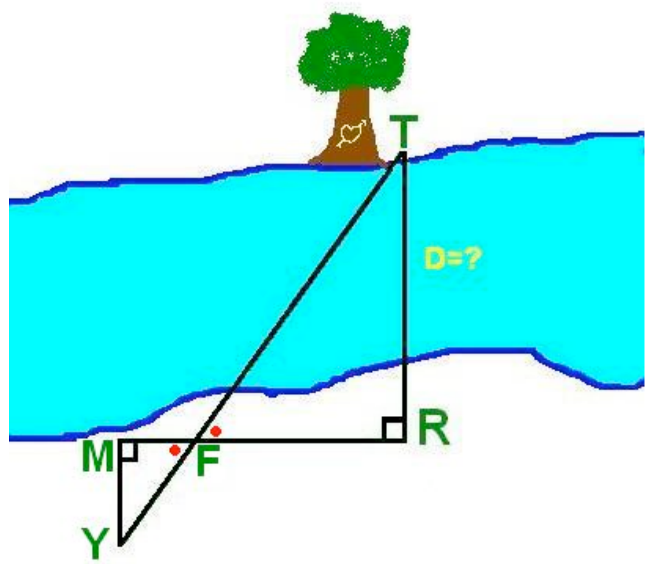
The only pre-knowledge you should know is how to solve a ratio. Whenever we have two ratios that are equal, then we can cross-multiply and the resulting products will always be equal. Observe:

If $\frac{a}{b} = \frac{c}{d}$**, then** $a \times d = b \times c$, and if we have three of the variables known, then there is a short-cut that allows us to find the remaining variable in only one step. Here is an example:

$\frac{5}{9} = \frac{13}{d}$**, then** $d = \frac{9 \times 13}{5}$**, or** $d = 23.4$ The short-cut is that whatever number is diagonally opposite the unknown "d", will appear on the denominator of the answer. The other two numbers will be multiplied and appear on the numerator of the answer.

The two main rules of similar triangles is that: (a) if two triangles have an AA relationship (two angles of one equal to two angles of the other), then they have an AAA relationship (all three angles of one, equal the three corresponding angles of the other). (b) If two there is an AAA relationship between two triangles then the triangles are similar and their sides are in the same ratio.

Let's see the ***Boy Scout's Method of Finding the Distance Across a River***, and then we will see how these rules above apply. Using the picture on the right, a Rock "R" is placed opposite a Tree "T" on the other side of the river. One then walks at right angles to the line TR along the river bank. A second marker "F" is placed after a certain distance FR. Continue walking another distance FM, at this point "M", turn at right angles to the left and walk away from the river, until You "Y", the marker at "F" and the Tree "T" all line up. Measure distance MY. So let us say, FR is 10 metres and FM is 3 metres, and MY is 5.76 metres. The two right angles are equal, and the two angles indicated by red dots are equal because they are vertically opposite angles. Hence we have an AA and therefore, an AAA, relationship and the triangles are similar. The distance TR can then be worked out as shown below:

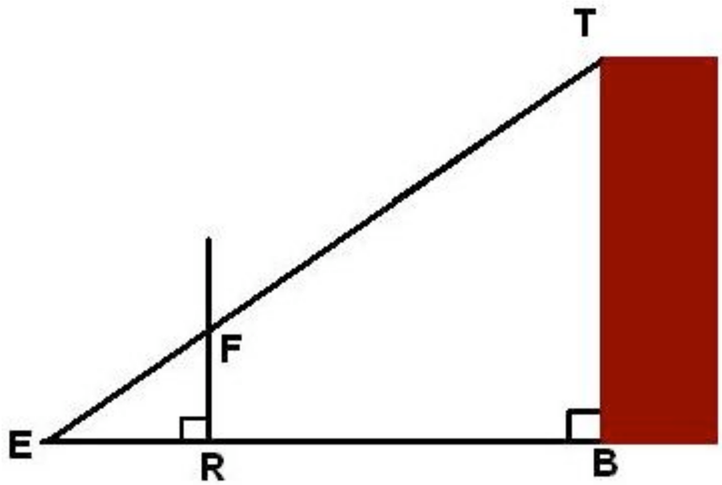


$\frac{RF}{FM} = \frac{RT}{MY}$**, therefore** $\frac{10}{3} = \frac{RT}{5.76}$ and using our shortcut:

$RT = \frac{10 \times 5.76}{3}$**, therefore** $RT = \frac{57.6}{3} = 19.2$ and the distance across the river is 19.2 metres.

Now, on the next page we will use similar triangles to find heights of a building (or a tree, etc).

As before, we will show you the ***Boy Scout's Method For Finding Heights*** first, and then illustrate with actual numbers. Observe the diagram below:



The Top of the building is "T", and its Bottom is "B". Measure out a distance from the Bottom and place a Rock "R" a known distance away. Now, measure out another distance RE. At point "R", have your assistant put a long stick at right angles to the ground. (We used our scout staves, to do this). Now, get down on your hands and knees and place your Eye on the ground at point "E". Have your assistant mover their fingers up the stick until your eye "E", their Fingers "F" and the Top of the building "B", line up. Lets say you measure RB to be 11 metres and ER = 2.5 metres, and RF to be 1.9 metres.

There are two triangle inside each other, a small one Δ EFR, and a large one, Δ ETB. They both have the same angles at "E", and they each have a right angle, therefore they have an AA and hence an AAA relationship and are therefore similar triangles. Also note that the distance RB does not come into the working of the problem because RB does not belong to a triangle. Instead we have side EB = ER + RB = 2.5 + 11 = 13.5 metres. Here is the working:

$$\frac{RF}{ER} = \frac{TB}{EB}, \text{ therefore } \frac{1.9}{2.5} = \frac{TB}{13.5} \text{ and using our shortcut: } TB = \frac{1.9 \times 13.5}{2.5}, \text{ or } TB = 10.26$$

Thus, the height of the building was 10.26 metres. Now, when I used this method at Boy Scout Camp, Canada was still using feet and inches. We were taught to do the problem by making RB a distance of 11 scout staves and ER a distance of 1 scout stave. We then measured the distance RF in inches, and automatically called the result "feet" and we had the height of the building or tree. So, if RF was 63 inches, the building was 63 feet! Magical, huh?? The reason this works is that the distance EB = ER + RB = 1 + 11 = 12. And the ratio of the small triangle to the large triangle is 1 : 12, the same ratio as feet to inches.

There are other methods using similar triangles as well, that I will try to add later.