A Neat Trick When Reversing Digits

The other day, I was trying to figure out how much older I am than my son. I was born in 1947, and he was born in 1974, so I had to subtract 74 – 47 which I got instantly as 27 using the trick below.

If you take a two digit number, such as 84 and reverse the digits, you get 48. Now subtract the two results and you get 36. This number, 36, is a multiple of 9, in fact it is 4 x 9, and if you subtract the digits of 84, you get 8 – 4 = 4. This is not a fluke, you always get a multiple of 9 that is equal to 9 times the difference between the two digits of your original number. In doing this, I will always subtract the smaller from the larger. Let's try another one, 93, whose reversed digit form is 39. Subtract them, I get 54, which is 9 x 6. This is therefore a multiple of 9, and the two digits of 93 are 6 apart, and 54 = 6 x 9.

Here is why it works. Any 2 digit number can be written in the form of 10T + U, where "T" is the Ten's digit and "U" is the Unit's digit. If I reverse the digits, I get a number that is equal to 10U + T. Let's subtract them:

$(10T + U) – (10U + T) = 10T + 1U – 10U – 1T = 9T – 9U$, or factoring we get: $9(T – U)$. Well this is just 9 times the difference between the digits "T" and "U". So, it is therefore a multiple of 9.

Is there a similar trick for three digit numbers? Well, they are in the form of 100H + 10T + U where "H" is the Hundred's digit and "T" and "U" are as described above. Reversing them, I get 100U + 10T + H. Now subtracting the two I get: $(100H + 10T + U) – (100U + 10T + H) = 100H + 10T + 1U – 100U – 10T – 1H$ or $99H – 99U$ and factoring, I get $99(H – U)$. So it is a multiple of 99 and equals 99 times the difference between the Hundreds digit and the Unit's digit.

Thus if I have 793 and reverse it I get 397, subtracting 793 – 397 should give me 99 times (7 – 3) or 99 x 4. To multiply by 99 quickly, I break it up into 11 x 9 and use my multiplying by 11 trick (See Saturday, March 7, 2009). So 99 x 4 = 11 x 9 x 4 = 11 x 36 = 396. Sure enough, when I subtract 793 – 397 I get 396.

This could be extended for four digit and five digit etc., and I leave it to the reader to explore and find their own tricks for this.