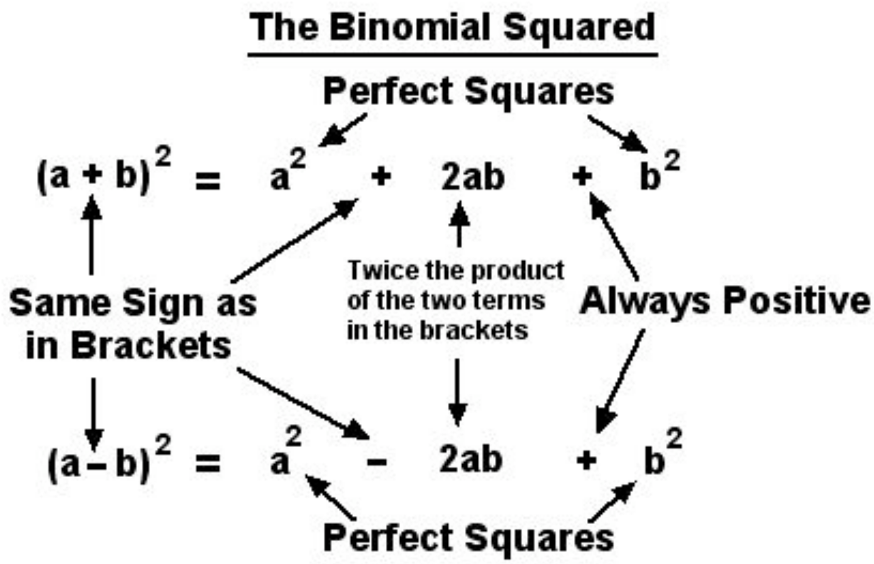
<u>Factoring a Binomial Square and A Math Trick Using It</u>

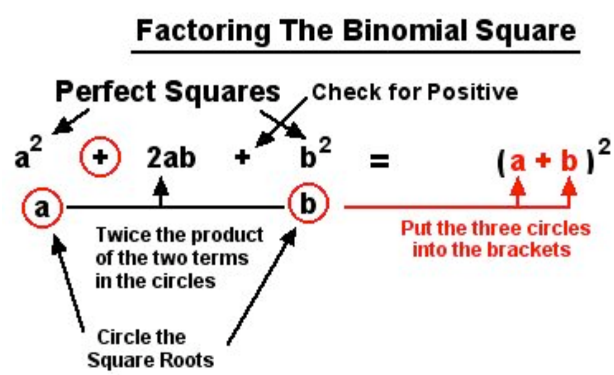
When you take $(a + b)^2$ or $(a - b)^2$ you get an interesting pattern as shown below with the following diagram.



Do you notice the first and the last term end up as perfect squares? That, if you take the two terms in the brackets, multiply them, and then double the product, you get the middle term? That the last term always ends up being positive (negative x negative or positive x positive)? That the sign of the middle term is the same as the sign in the bracket? Thus, if I had $(3x - 5y)^2$, then I would:

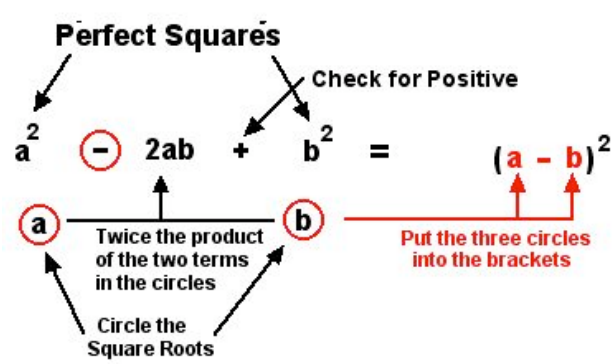
(1) Square 3x to get $\mathbf{9x^2}$, and square – 5y to get $\mathbf{+ 25y^2}$, and
(2) Multiply 3x times – 5y to get – 15xy, and double it to get **– 30xy**, and
(3) Put it altogether to get $(3x - 5y)^2 = \mathbf{9x^2 - 30xy + 25y^2}$

Now, if we reverse the procedure, we get what is called "factoring". First of all we need to check that the given trinomial is a perfect square trinomial, then we shall write the answer. This is all illustrated in the diagram on the right.



Things to notice:

(1) Check that first and last terms are perfect squares, put their square roots in circles below them.
(2) Check that the sign of the last term is a positive or plus sign.
(3) Multiply the two square roots in the circles and then double your result. Now check that this equals the middle term. If all three checks work out, then circle the sign of the middle term.
(4) Now to factor, put the three things in your three circles into the bracket of your answer, and attach a squared sign to your bracket.



Now on the next page we will look at three examples of this factoring.

Three examples of factoring a perfect square trinomial.

(1) Factor $4x^2 + 20x + 25$. Instead of circling, I'll put the results in red brackets.
First check off, $4x^2$ is a perfect square whose square root is **(2x),** 25 is a perfect square whose square root is **(5).** Now, check that the third term has a positive or plus sign in front of it. Yes it does. Now multiply the two square roots and double your answer: **(2x) times (5) = 10x, doubled equals 20x**. Now check that this equals the middle term. It does, so circle the sign of the middle term **(+)**. Your answer now is the three circles (brackets) put in a bracket with a squared sign on it: $(2x + 5)^2$.

(2) Factor $9r^2 - 42rt + 49t^2$. Instead of circling, I'll put the results in red brackets.
First check off, $9r^2$ is a perfect square whose square root is **(3r),** $49t^2$ is a perfect square whose square root is **(7t).** Now, check that the third term has a positive or plus sign in front of it. Yes it does. Now multiply the two square roots and double your answer: **(3r) times (7t) = 21rt, doubled equals 42rt**. Now check that this equals the middle term. It does, so circle the sign of the middle term **(–)**. Your answer now is the three circles (brackets) put in a bracket with a squared sign on it: $(3r - 7t)^2$.

(3) Factor $100y^2 + 90y - 81$. Instead of circling, I'll put the results in red brackets.
First check off, $100y^2$ is a perfect square whose square root is **(10y),** 81 is a perfect square whose square root is **(9).** Now, check that the third term has a positive or plus sign in front of it. **No it doesn't**. Now multiply the two square roots and double your answer: **(10y) times (9) = 90y, doubled equals 180y**. Now check that this equals the middle term. No it doesn't, so this fails the pattern on two different checks. It is not q perfect square and cannot be factored this way, try it using factoring the hard trinomial, if it fails that, then it can not be factored.

Now, what do you know about the following numbers: 961, 1681 and 2601?

961, starts and ends with perfect squares, 9 and 1, whose square roots are **(3)** and **(1)** respectively. If I multiply the 3 times the 1 and double it, I get "6", the middle, or ten's digit. Hence, 961 is a perfect square whose square root is **31**.

1681, starts and ends with perfect squares, 16 and 1, whose square roots are **(4)** and **(1)** respectively. If I multiply the 4 times the 1 and double it, I get "8", the ten's digit. Hence, 1681 is a perfect square whose square root is **41**.

2601, can be written as 25 hundred, 10 tens and 1 unit. This starts and ends with perfect squares, 25 and 1, whose square roots are **(5)** and **(1)** respectively. If I multiply the 5 times the 1 and double it, I get "10", the ten's digit after I borrowed one from the 26. Hence, 2601, or $25^{\mathbf{10}}1$ is a perfect square whose square root is **51**.

Something that I always do is look at all numbers and try to analyze them. This is an example of that.