Using Cross-Border Shopping to Solve "Type 2" Equations

I define a "type 2" equation as an equation where the variable, or unknown, appears more than once on both sides of the equation. Yesterday, Tuesday, July 28, 2009, I showed how you can "move" the terms across the equals signs by adding or subtracting the opposites. So the example I showed was:

3A – 5 = A + 9 which I immediately changed into 3A – 5 = 1A + 9. I then added 5 to each side:
(1) 3A – 5 + 5 = 1A + 9 + 5 and reduced to get:
(2) 3A = 1A + 14. I then subtracted 1A from each side to get:
(3) 3A – 1A = 1A – 1A + 14. Then I simplified to get:
(4) 2A = 14. I then had a "type 1" equation, dividing both sides by 2 I got:
(5) $\frac{2\boldsymbol{A}}{2} = \frac{14}{2}$ and then simplifying again, I got:
(6) A = 7

One can think of the equals sign as a "border" between two countries, with one country being the land of the "A's" where terms with the variable "A" live, and the other country being the land of the "numbers only" where terms that are only numbers live. Living on the equals sign is Joe the border guard. His only job, is to change the sign of anything that crosses his border to the opposite. Addition becomes subtraction, subtraction becomes addition, multiplication becomes division, and division becomes multiplication. Also, roots become powers and powers become roots. Observe this in action with the same question as we used above:

3A – 5 = 1A + 9
(1) 3A – 1A = 9 + 5. Notice the terms in blue have changed sides, so their signs in front are opposite.
(2) 2A = 14. Simplify.
(3) $\boldsymbol{A} = \frac{14}{2}$. The "2" changed sides so it changed from multiply to divide.
(4) A = 7. Simplifying yields the same answer as above. A typical "type 2" equation now takes only 4 steps, no matter how many terms and, in fact, steps 3 and 4 can be combined.

Observe the next one:

Solve: 9B + 3B – 5 + 7 + 5B = 4B – 6 + 8 + 2B
(1) 9B + 3B – 4B – 2B + 5B = – 6 + 8 + 5 – 7 Notice the terms in blue have changed sides AND signs
(2) 6B = 0. Simplify
(3) $\boldsymbol{B} = \frac{0}{6} = 0$

And one more example of "cross – border shopping" :

Solve $\frac{3\boldsymbol{w}}{5} - 6 = 10$

(1) $\frac{3\boldsymbol{w}}{5} = 10 + 6 = 16$. Moving the "– 6" across the border, changing the sign, and simplifying.
(2) 3w = 16 x 5 = 80. Moving the "divide by 5" across the border and making it "multiplying by 5".
(3) $\boldsymbol{w} = \frac{80}{3} = 26\frac{2}{3} = 26.\bar{6}$. Moving the "multiply by 3" across the border and making it "divide by 3".

Now, steps 2 and 3 could also be combined by thinking of $\frac{3w}{5}$ as "w is multiplied by 3 and divided by 5" so when it crosses the border, it must become "multiply by 5 and divide by 3", so step (2) could now look like:

(2) $w = \frac{5}{3} \times 16 = 26\frac{2}{3} = 26.\bar{6}$. So, to get rid of a fraction in front of a variable, move it to the others side and multiply by the reciprocal.

Now let's see the same idea with powers and roots.

Solve $x^4 = 38$

(1) $x = \sqrt[4]{38} = 2.482823796$

Note, "power of 4", changes to the "4th root" when it crosses the border.

and solve: $\sqrt{2x} = 9$

(1) $$\begin{aligned} 2x &= 9^2 = 81 \\ x &= \frac{81}{2} = 40.5 \end{aligned}$$

Note. The "square root, or 2nd root" changes to "the square or the power of 2" when it crosses the border.

Finally if we involve a power and a root as in this question:

Solve: $\sqrt[3]{4x}^5 = 19$

(1) $\sqrt[3]{4x}^5 = (4x)^{\frac{5}{3}} = 19$, change from radical to exponential form, then:

(2) $4x = 19^{\frac{3}{5}} = 5.851297242$. Change the power to its reciprocal as it "crosses the border"

(3) $x = \frac{5.851297242}{4} = 1.462824311$

So, if you look at all the examples on the last two pages, you can see that there is a pattern that can be summed up in one simple statement: "If you cross the border, you change the sign". This is the 5th big picture idea that I have on the "Ten Big Picture Ideas" poster that I have in the download section.