

Checking Your Work by Casting Out Nines

The digital sum of a number is a one digit result by adding up the digits of a number. If the number obtained by adding up the digits has more than one digit, then add those digits. Continue doing this until you get a result from 1 to 9. For example: 7 296 has a digital sum of $7 + 2 + 9 + 6 = 24$, and $2 + 4 = 6$. Thus its digital sum is considered to be 6.

Now look again at the number 7 296 and cancel out any digit that is a “9” or that add up to “9”. So we would cancel out the 7 and the 2 (since they add up to 9) and the 9. This looks like this: ~~7~~~~2~~~~9~~6. Notice we end up with answer of 6, which is the same as the digital sum. Look at this one 4 005 996. Canceling (or casting out) the nines and digits that add up to 9 leave us with ~~4~~~~0~~~~0~~~~5~~~~9~~~~9~~6, or a digital sum of 6 as well. Adding all the digits we would have $4 + 0 + 0 + 5 + 9 + 9 + 6 = 33$ and $3 + 3 = 6$. So, it works, and a “0” and a “9” do not change the result. If all digits get cast out as in the number 5 397 426 we get ~~5~~~~3~~~~9~~~~7~~~~4~~~~2~~~~6~~ or a digital sum of “0”. We will see below that in this scheme of things, $0 = 9$.

Now why are we looking at this? Well it just so happens that what ever you do to numbers (add, subtract, multiply or divide) can be checked by doing the same things to their digital sums. And, of course, the quickest way to get the digital sum is to cast out the 9’s. Observe:

Addition:	Digital Sum	Subtraction	Digital Sum	Multiplication	Digital Sum	Division	Digital Sum
5892	6	34953	6	34578	0	23) 35684	= 8
3728	2	<u> - 2545</u>	7	<u> x 134</u>	<u> x 8</u>		
497	2	32408	8	138312	0		
<u> 1063</u>	1			<u>103634</u>	8		
11180	2			<u> 34578</u>	0		
				4632452	8	Ans : 1551	3
						Rem : 11	2

In the addition example, the digital sum of 11 180 is 2 and the sum of the digital sums is 11, and $1 + 1 = 2$. So the question is probably correct. There is a slight chance (transposing 2 digits) that the digital sums are correct but the sum is NOT correct. However, if the digital sum is NOT correct then we know that the original question was done wrong. This assumes we did the digital sums correctly!

In the subtraction example, the digital sum of 34 953 equals “6”. The digital sum of 2 545 = 7, and since we can’t subtract 7 from 6 without going into the negatives, we add 9 onto the “6” to get 15. Now subtracting $15 - 7 = 8$, so the result is probably correct. If a digital sum was 0, and a negative would result, then add 9 to 0 to get 9 and then it will work out.

In the multiplication example, the digital sum of the answer should be “0” since if we multiply the digital sums of the two numbers multiplied we get $0 \times 8 = 0$. I also took the digital sums of the red “partial” sums and see that they are not all “0” as they should be since I am multiplying by first 4, then 3, then 1 by a number whose digital sum is equal to “0”. So thus it tells me the answer is wrong, and that the error appeared in the middle number when I did 34578×3 and got 103634. In fact the result should have been 103734. This has a digital sum of “0” and the answer would then change to 4633452, whose digital sum is “0” as well.

To check the division problem we would normally multiply $23 \times 35\ 684$ and add on the remainder of 11. So we will do the same thing with the digital sums: Digital sum of 23 = $2 + 3 = 5$. Digital sum of our answer 1 551 is $1 + 5 + 5 + 1 = 12$ and $1 + 2 = 3$. Digital sum of our remainder of 11 is $1 + 1 = 2$. And the digital sum of our dividend 35 684 is 8 since the other digits can be “cast out”. Now we check $3 \times 5 + 2$ and get 17, whose digital sum is also “8”. So our problem is probably correct.