How to Multiply and Divide Rational Expressions

Yesterday, we looked at how to Multiply and Divide fractions. Since rational expressions are just fractions, the rules are exactly the same. Therefore, we multiply the tops (numerators) and we multiply the bottoms (denominators).
Below, I have a table that compares the two.

Fraction	Rational Expression
$\frac{3}{7} \times \frac{2}{5} = \frac{3 \times 2}{7 \times 5} = \frac{6}{35}$	$\frac{3a}{7b} \times \frac{2a^2}{5} = \frac{3a \times 2a^2}{7b \times 5} = \frac{6a^3}{35b}$

Now, there is also a rule in multiplying fractions (rule 10 of 10) that if there is a number on the top of the fraction that has a factor in common with a number on the bottom of the fraction, you can divide the numerator and the denominator by that common factor. Here is an example:

$$\frac{12}{7} \times \frac{21}{10} \times \frac{15}{6} = \frac{6 \times \cancel{2}}{\cancel{7}} \times \frac{3 \times \cancel{7}}{\cancel{2} \times 5} \times \frac{\cancel{3} \times \cancel{5}}{2 \times \cancel{3}} = \frac{6 \times 3}{2} = \frac{18}{2} = 9$$

Quite often, the second step, where the numbers are factored, is left out. However, with rational expression it is really important that THE FIRST STEP IS TO FACTOR numerator and denominator. Here is an example:

$$\frac{x^2 + 6x - 16}{x^2 - 64} = \frac{(x + 8)(x - 2)}{(x + 8)(x - 8)} = \frac{(x - 2)}{(x - 8)} = \frac{x - 2}{x - 8}$$

Now, a few things to note about the above example. I FACTORED FIRST. (You CANNOT cross out the the two x^2's in the first step and the 16 and 64 in the first step because of the plus and minus signs. You can cross out things that are the same that are MULTIPLIED not added or subtracted. Secondly, I cross out two identical brackets, one on the top and one on the bottom, IF THEY ARE IDENTICAL. Thus, the (x + 8) bracket can cancel out another (x + 8) bracket, but not a (x – 8) bracket. In the final answer, the brackets can be dropped since the long divide bar is like a bracket. Finally, in your final answer, DO NOT CANCEL the x's or divide the 2 into the 8, because of the subtract signs. If they were multiplied, I could.

Now let's look at an example of multiplying two rational expressions where we will FACTOR FIRST.

$$\frac{3a^2 + 14a - 5}{2a^2 - 2} \times \frac{4a^2 + 8a + 4}{15a - 5} = \frac{(3a - 1)(a + 5)}{2(a + 1)(a - 1)} \times \frac{4(a + 1)(a + 1)}{5(3a - 1)} = \frac{2(a + 5)(a + 1)}{5(a - 1)}$$

Note, in the final answer we do NOT cancel (a + 1) bracket on the top with the (a – 1) bracket on the bottom, since they are not identical. Also, we do NOT cancel the 5 on the top inside a bracket with the 5 on the bottom outside the bracket. They must both be outside of the bracket to cancel, as we did in the middle step where we divided the 2 into the 4, since they were both outside of brackets.

One other thing about canceling, since (a + 7) = (7 + a) they can be cancelled if one is in the numerator and one is in the denominator. Also, (a – 7) = -1 x (7 – a), so if you cancel them (assuming one is in the numerator and one is in the denominator) you will be left with a negative 1 for one of them. I'll illustrate this below.

Let's see how we divide rational expressions. Look at the example below, and remember the rule discussed yesterday, where we multiply by the reciprocal of the second term.

Fraction	Rational Expression
$\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \times \frac{5}{3} = \frac{2 \times 5}{7 \times 3} = \frac{10}{21}$	$\frac{5m-25}{2m^2+9m+7} \div \frac{25-5m}{m+1} = \frac{5m-25}{2m^2+9m+7} \times \frac{m+1}{25-5m} =$ $\frac{5(m-5)}{(2m+7)(m+1)} \times \frac{(m+1)}{5(5-m)} = \frac{-1}{2m+7}$

Note, on the right above, we changed the divide sign to a multiply sign and flipped over the SECOND term (to make its reciprocal). Finally, when we cancelled, the 5's cancelled because they were both OUTSIDE brackets, and the (m – 5) bracket cancelled the (5 – m) bracket, BUT LEFT A NEGATIVE ONE.

So there you have it, rational expressions are just fractions, so they follow the rules of fractions. I feel it is really useful for the reader to look at Monday's and Tuesday's sessions (April 27, 28) in order to solidify in your own mind, the rules of fractions.