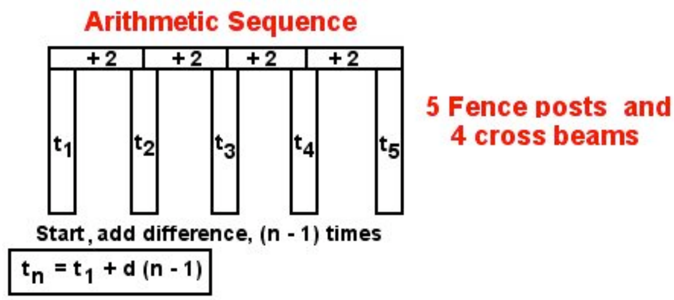
Formulas Involved with Sequences and Series

A sequence is a list of numbers, or other things, usually, but not necessarily, with some sort of logic to it. If the terms of the sequence are added, or subtracted, like 1 + 3 + 5 + 7 + 9 +… then we have what is called a series.

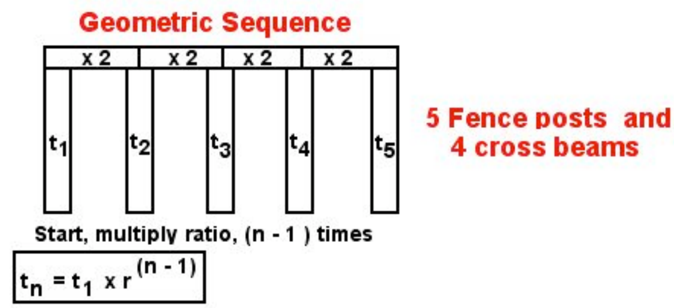
The first two formulas involve finding a term in an arithmetic or geometric sequence.

Arithmetic Sequence/Series	What the Formula Computes	Geometric Sequence/Series
$t_n = t_1 + d(n-1)$	Find a certain term (t_n)	$t_n = t_1 \times r^{(n-1)}$

It is important to understand where formulas come from. I call the two formulas above, the fence-post formulas. Observe the diagram below right.



The number of cross beams is always one less than the number of fence posts. For the arithmetic sequence, the number of “differences (+ 2)” you are adding is one less than the number of terms. For the geometric sequence, the number of “ratios (x 2)” you are multiplyinging is one less than the number of terms.



The formulas have the same pattern, change the “ + “ sign to a “ x “ sign, the word “difference” to “ratio”, and the bracket (n – 1) from multiplied to an exponent.

Here are two problems where you are asked to find the value of a term.

(1) 5, 8, 11, 14, 17, … Find the 38th term.

(2) 3, 6, 12, 24, 48, … Find the 16th term.

Arithmetic Sequence/Series	Suggested Steps	Geometric Sequence/Series
$t_1 = 5$ $d = +3$ $n = 38$ $t_n = t_{38} = ?$	Write out the variables and what you know about them	$t_1 = 3$ $r = +2$ $n = 16$ $t_n = t_{16} = ?$
$t_n = t_1 + d(n-1)$	Choose the correct formula	$t_n = t_1 \times r^{(n-1)}$
$t_n = 5 + 3(38-1)$	Fill in the formula	$t_n = 3 \times 2^{(16-1)}$
$t_n = 5 + 3(37)$ $t_n = 5 + 111$ $t_n = 116$	Solve for t_n	$t_n = 3 \times 2^{(15)}$ $t_n = 3 \times 32\ 768$ $t_n = 98\ 304$

A few notes on the above. I really find it useful to write out the variables involved, and then put down the values of those that I know. Note, the order of operations is involved: brackets, exponents, then multiplying or dividing, then adding or subtracting. Some books will use the mnemonic BEDMAS to remember this. Also some books will put the formula for arithmetic sequence in this order: $t_n = t_1 + (n-1)d$. And some books will use “a” for the first term instead of “t_1”. Since I am trying to illustrate the pattern between the two formulas, I have done it the way I showed you above.

Finally, for sequences, since there are four variables used, a question could be asked about finding any of the variables, given the other three. My second set of two sequence questions will illustrate this.

(1) Find the four arithmetic means between 13 and 84.
(2) Find the four geometric means between 7 and 138.

Here are the solutions:

Arithmetic Sequence/Series	Suggested Steps	Geometric Sequence/Series
13 ____ ____ ____ ____ 84 t_1 t_2 t_3 t_4 t_5 t_6	Draw the diagram of the terms	7 ____ ____ ____ ____ 138 t_1 t_2 t_3 t_4 t_5 t_6
$t_1 = 13$ $d = ?$ $n = 6$ $t_n = t_6 = 84$	Write out the variables and what you know about them	$t_1 = 7$ $r = ?$ $n = 6$ $t_n = t_6 = 138$
$t_n = t_1 + d(n - 1)$	Choose the correct formula	$t_n = t_1 \times r^{(n-1)}$
$84 = 13 + d(6 - 1)$	Fill in the formula	$138 = 7 \times r^{(6-1)}$
$84 - 13 = d5$ $71 = 5\boldsymbol{d}$ $\frac{71}{5} = \boldsymbol{d}$ $14.2 = \boldsymbol{d}$	Solve for "d" or "r"	$\frac{138}{7} = \boldsymbol{r}^5$ $19.7142857 = \boldsymbol{r}^5$ $\sqrt[5]{19.7142857} = \boldsymbol{r}$ $1.347342796 = \boldsymbol{r}$
13, 27.2, 41.4, 55.6 69.8 84 t_1 t_2 t_3 t_4 t_5 t_6	Fill in the blanks by adding 14.2 or by multiplying by 1.81533261.	7 12.71, 23.07, 41.88, 76.02 138 t_1 t_2 t_3 t_4 t_5 t_6
69.8 + 14.2 = 84 √	← Check by adding 14.2 + 69.8 to see if you get 84	The numbers above have been rounded off to two decimal places
	Check multiplying 76.01912685 x 1.81533261 to see if you get 138 →	76.01912685 x 1.81533261 = 137.99999999 or 138. √

You may have seen the words "arithmetic mean" before. It is sometimes called the average. If I was to find the ONE arithmetic mean between 13 and 84, then I would be looking for the average and I could say (13 + 84) ÷ 2 = 97 ÷ 2 = 48.5, and the answer pops out really quickly. So the formula is : **average** $= \frac{t_1 + t_3}{2}$.

Is there a similar formula for finding the ONE geometric mean between 7 and 138? Indeed there is, just substitute " x " for " + " and square root for " ÷ 2" and you get: **Single Geometric Mean** $= \sqrt{t_1 \times t_3}$.

Solving this we get the single geometric mean between 7 and 138 as $\sqrt{7 \times 138} = \sqrt{966} = 31.08054054$

Finally, find the eleven geometric means between 1 and 2, gives you the following diagram:

1 ____ ____ ____ ____ ____ ____ ____ ____ ____ ____ ____ 2
t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10} t_{11} t_{12} t_{13}

Using the above techniques we can get the common ratio as **Common ratio** $= \sqrt[12]{2} = 1.059463094$

You bumped into this number two weeks ago, on Wednesday, May 13, 2009. It is the ratio of the notes of the western musical scale! Between one note " C " and the note "C, the octave above", the ratio is 1 : 2 and there are 12 gaps in between.!