A Fibonacci Math Trick

(and why it works)

A Fibonacci sequence is a sequence where we start with any two numbers as the first two terms (t_1 and t_2) and add these to get the next term (t_3). Then term 2 and term 3 are added to get term 4, term 3 and term 4 are added to get term 5, and so on. We call this a recursive sequence and its formula would be $t_n = t_{n-1} + t_{n-2}$ which means "to find a term, add the term before it , and the term two before it". Here are some examples.

1, 1, 2, 3, 5, 8, 13, 21 ,34, 55, 89, 144, …

8, 9, 17, 26, 43, 69, 112, 181, …

-5, -2, -7, -9, -16, -25, …

And from yesterday: $2, 1 + \sqrt{5}, 3 + \sqrt{5}, 4 + 2\sqrt{5}, 7 + 3\sqrt{5}, 11 + 5\sqrt{5}, \ldots$

So as you can see, you can start with ANY two numbers, positive, negative, rationals, irrationals, etc.

So, here is today's trick. Ask a group of people to each start with any two numbers, these will be term 1 and term 2. They should maybe have a calculator with them so that they can make sure their terms are correct, for this is important. Ask them to form a Fibonacci sequence with their two numbers until they get to term 10. They will now have a list of ten numbers as seen with two examples below. Using a calculator, ask them to add up their 10 numbers, but **keep the answer hidden**. You then go around to each person, look at their ten numbers, then within seconds write down their totals, no calculator, no pencil and paper, all done in your head instantly!

Term #	Number
1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21
9	34
10	55
Total	

Term #	Number
1	8
2	9
3	17
4	26
5	43
6	69
7	112
8	181
9	293
10	474
Total	

So, looking at the two examples above, you instantly write down 143 as the total for the left hand example and 1 232 as the total to the right hand example. Here is how it works. The total of the first ten Fibonacci numbers is 11 times the seventh term! Now on March 7, I showed a math trick for multiplying any whole number by 11, instantly in your head. Here is the link: http://www.mathhiker.com/archives/258. Sure enough 11 x 13 = 143 and 11 x 112 = 1 232. It seems to work, but why?

See the next page for the proof:

Proof of why the Fibonacci trick works.

Let's look at the ten terms using variables:

Term #	Number	
1	1x	term 1
2	1y	term 2
3	1x + 1y	term 1 plus term 2
4	1x + 2y	term 2 plus term 3
5	2x + 3y	term 3 plus term 4
6	3x + 5y	term 4 plus term 5
7	**5x + 8y**	term 5 plus term 6
8	8x +13y	term 6 plus term 7
9	13x + 21y	term 7 plus term 8
10	21x + 34y	term 8 plus term 9
Total (all 10 terms)	55x + 88y	= 11(**5x + 8y**), which of course, is 11 times the seventh term!

So, using a bit of algebra, adding like terms and factoring, you can see why this trick works.