The Fibonacci Sequence and the Golden Ratio

The Fibonacci sequence was first discussed by Leanardo Fibonacci, an Italian Mathematician who lived about 1170 AD to 1230 AD. There will be more about him on Thursday's History Day (May 20, 2009). The thing I like about the Fibonacci Sequence is, although simple in development, it has astounding patterns and uses. Some of them may bring in over $1 000 000 or more! Wonder how, read on.

The Fibonacci numbers show up in nature (sun flower swirls, pine cones, branches around a tree etc). I have a book called the "Elliott Wave Theory" by W.D. Gann and R. N. Elliott, where they say the Fibonacci sequence appears in stock fluctuations. The classic Elliot Wave series consists of an initial wave up, a second wave down (often retracing 61.8% of the initial move up), then the third wave (usually the largest) up again, then another retrenchment, and finally the fifth wave, which would exhaust the movement. In addition, each of the major waves (1, 3, and 5) could themselves be separated into subwaves, and so on, and exhibit other Fibonacci relationships. So you might be able to make $1 million on that. Or you could write a book, as Dan Brown did, named "The DaVinci Code", that talks about the Fibonacci sequence.

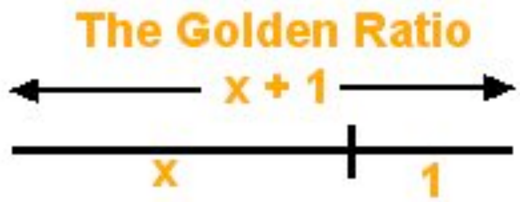
One neat thing that happens with the Fibonacci sequence, no matter which two numbers that you start with, is that as you take each number and divide it by the number before it, you get closer and closer to the golden ratio (φ = 1.618033989, pronounced "phi" or "fie"). See the spreadsheet below where I illustrate this.

Term Number	Term	Ratio	**Term Number**	Term	Ratio	**Term Number**	Term	Ratio
1	1		1	5		1	-14	
2	1	1	2	12	2.4	2	18	-1.28571
3	2	2	3	17	1.416667	3	4	0.222222
4	3	1.5	4	29	1.705882	4	22	5.5
5	5	1.666667	5	46	1.586207	5	26	1.181818
6	8	1.6	6	75	1.630435	6	48	1.846154
7	13	1.625	7	121	1.613333	7	74	1.541667
8	21	1.615385	8	196	1.619835	8	122	1.648649
9	34	1.619048	9	317	1.617347	9	196	1.606557
10	55	1.617647	10	513	1.618297	10	318	1.622449
11	89	1.618182	11	830	1.617934	11	514	1.616352
12	144	1.617978	12	1343	1.618072	12	832	1.618677
13	233	1.618056	13	2173	1.618019	13	1346	1.617788
14	377	1.618026	14	3516	1.61804	14	2178	1.618128
15	610	1.618037	15	5689	1.618032	15	3524	1.617998
16	987	1.618033	16	9205	1.618035	16	5702	1.618048
17	1597	1.618034	17	14894	1.618034	17	9226	1.618029
18	2584	1.618034	18	24099	1.618034	18	14928	1.618036
19	4181	1.618034	19	38993	1.618034	19	24154	1.618033
20	6765	1.618034	20	63092	1.618034	20	39082	1.618034

The Golden Ratio

You can see that, no matter what you start with the ratio of 1.618034 keeps magically appearing! Again this appears in nature a lot, ratios of knuckles, nose to head length, ratios of angles between consecutive stems on flowers, and so on. A 100 m rugby pitch 70 metres wide with end zones of 6.63 metres has a total length to width ratio of 1.618! That's why it is really the "Golden" game!

The definition of the Golden Ratio is taken from the following diagram:



The ratio of x to the whole, x + 1, must be the same as the ratio of 1 to x. We set up this ratio and solve it here:

$$\frac{x}{x+1} = \frac{1}{x}, \quad \textbf{Set up the ratio}$$

$$x^2 = 1(x+1), \therefore x^2 = 1x + 1, \quad \textbf{cross-multiply}$$

$$x^2 - 1x - 1 = 0, \quad \textbf{make right-hand side = 0}$$

Solve using the quadratic formula, a = 1, b = -1, c = -1

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{5}}{2}, \quad \textbf{or x = 1.618033989, or x = -0.618033989}$$

So, if we get any ratio that yields the equation $x^2 - x - 1 = 0$, or $x = \frac{1+\sqrt{5}}{2}$, then we have a Golden Ratio (**φ**).

Here are a couple of surprise examples:

The continued fraction: $x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}$ and the continued radical: $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$

If you look at the left hand fraction, the denominator or the fraction after the first plus sign is actually "x" itself.

Thus we could write the left hand fraction as $x = 1 + \frac{1}{x}, \therefore x = \frac{x+1}{x}$.

If I cross-multiply and make one side equal to 0, I get $x^2 - x - 1 = 0$.

This yields answers of x = **φ,** and x = - 1 / **φ.**

On the right hand side of above, the continued radical after the first plus sign is the same as "x", so the continued radical can be written as $x = \sqrt{1 + x}$. Square both sides and you get $x^2 = 1 + x$, and $x^2 - 1x - 1 = 0$.

So x = **φ** x = - 1 / **φ.**

Here are some other neat things about the Golden Ratio (**φ**)**= 1.618033989**

$$\frac{1}{\phi} = .618\ldots = \phi - 1$$

$$\phi^2 = 2.618\ldots = \phi + 1$$

More about this on Friday.

I urge readers to look at the following two websites, one all about **φ** (phi), http://goldennumber.net/ and the other, a website about Fashion and **φ.** http://thefashioncode.com/.