

What's the Probability of Getting Five Correct in a Row?

There are many applications where one is interested in seeing how many times in a row can one be successful. In business, one of the best examples is in quality control, where a company is interested in as few failures in manufacturing, or errors in accounting, or “non-successes” in any production. An example of this is the Motorola developed “Six Sigma” for quality control. They are aiming at 99.99966% perfection. For this post however, I will look at some areas closer to your life and concentrate on just getting 5 correct or 5 successes in a row. Some examples:

- (1) What is the probability of getting 5 free throws in a row in basketball?
- (2) What is the probability of getting 5 serves in tennis or volleyball in a row?
- (3) What is the probability of getting 5 true/false questions in a row correct?
- (4) What is the probability of getting 5 steps in a Math question correct?
- (5) What is the probability of making five passes in a row in a rugby backfield?

... and so on.

The fifth one above is very near and dear to me as I coached rugby for over 37 years.

Observe the following table: To find consecutive probabilities we will multiply each probability. This assumes that each attempt is independent of the other events, which in these cases is probably the case.

Prob. Level	1st attempt	2nd attempt	3rd attempt	4th attempt	5th attempt	Final Probability
50% or 0.5	0.5	0.5	0.5	0.5	0.5	$0.5^5 = .03125$ or 3.125%

Wow, if your chances of success on a free throw, or a volleyball serve, or a rugby pass is only 50%, then the chances of getting five in a row is only 3.125%. This is why, in rugby, where there usually is 5 passes to get the ball to your wingers (the fastest payers), at younger levels we quite often will play these players in the centres where it takes fewer passes for them to get it.

If your Math ability is weak at the grade 9 and grade 10 level, and you just get by with a 50% pass mark on most topics, then in Math 11, where there quite often are five steps to get the answer, you may get frustrated. You have to get through five 50% correct steps to get a correct answer! When I see students at this level, I then point out to them how useful it will be for them to put down ALL working on each step, so that part marks can be applied. These “part marks” will boost their letter grade at least one level over the year.

What can you do about this? Well there is a really neat solution for you. Let's say you practice your free throws, or your serve, or your factoring skills, or math skills so that you get a 60% level of success. Observe the following table:

Prob. Level	1st attempt	2nd attempt	3rd attempt	4th attempt	5th attempt	Final Probability
60% or 0.6	0.6	0.6	0.6	0.6	0.6	$0.6^5 = .07776$ or 7.776%

Wow, again! By only improving 10%, from 50% to 60%, we more than double our chances of getting 5 in a row. Hey, this is good, let's see what happens if I improve another 10% to 70%.

Prob. Level	1st attempt	2nd attempt	3rd attempt	4th attempt	5th attempt	Final Probability
70% or 0.7	0.7	0.7	0.7	0.7	0.7	$0.7^5 = 0.1681$ or 16.81%

Great, another “more than doubling”. It is now greater than five times more likely to occur than if I worked at 50% level! ($5 \times 0.3125 = 0.15625$). Let's look at some more:

Prob. Level	1st attempt	2nd attempt	3rd attempt	4th attempt	5th attempt	Final Probability
80% or 0.8	0.8	0.8	0.8	0.8	0.8	$0.8^5 = 0.32768$ or 32.77%
90% or 0.9	0.9	0.9	0.9	0.9	0.9	$0.9^5 = 0.59049$ or 59.05%
95% or 0.95	0.95	0.95	0.95	0.95	0.95	$0.95^5 = 0.7738$ or 77.38%
99% or 0.99	0.99	0.99	0.99	0.99	0.99	$0.99^5 = 0.9510$ or 95.10%

There, you are. You need to be at the 90% on your skills, in order to get near 60% chance of 5 in a row. To pass, that is get over 50% for 5 in a row, you need to be at the 87.06% level for each step.