Introduction to Rational Expressions

What is a rational expression? Why, it is merely a fraction, where the top (numerator) and/or the bottom (denominator) contain variable(s) as well as numbers. Since they are fractions, they follow all the rules of fractions that will see on the Monday and Tuesday sessions this week and the next few weeks. I urge you to look at Monday and Tuesday sessions this week to make sure you remind yourself what some of those rules are, and maybe, for the first time, understand why those rules work.

Here are some examples of rational expressions:

$$\frac{2n}{5},\ \frac{x+3}{5x},\ \frac{p^2-2p+5}{3x-5},\ \frac{n^2-3n-40}{n^2-25},\ \frac{2}{5k}$$

Today we look at the first two rules that involve rational expressions, and they are the same rules we saw yesterday with fractions.

(1) Whatever you multiply the numerator by, you must also multiply the denominator by the same thing. This is used if you want to have the denominator changed when adding or subtracting rational expressions. Here is an example: $\frac{2x+5}{4} = \frac{2x+5}{4} \times \frac{3x}{3x} = \frac{(2x+5)3x}{(4)3x} = \frac{6x^2+15x}{12x}$. Notice I multiplied both the numerator and the denominator by the same thing, 3x.

(2) Whatever you divide the numerator by, you must also divide the denominator by the same thing. This is a very powerful technique for simplifying rational expressions. Here are a few examples:

(a) $\frac{15x+25}{5} = \frac{5(3x+5)}{5} = \frac{\cancel{5}(3x+5)}{\cancel{5}} = 3x+5$

Notice, that the first step is to FACTOR both the numerator and the denominator FIRST. Then look at canceling common factors, in case the 5 on the top with the 5 on the bottom. So, I am really dividing the top by 5 and dividing the bottom by 5, the SAME THING.

(b) $\frac{n^2-3n-40}{n^2-25} = \frac{(n-8)(n+5)}{(n-5)(n+5)} = \frac{(n-8)}{(n-5)}$

Same strategy here, FACTOR FIRST, then cancel the common factors of (n + 5) on the top and (n + 5) on the bottom. Again here I am dividing the top and the bottom by the same thing, (n + 5).

(c) Substitute r = 14.671 into the following: $\frac{12r^2-44r+40}{12r-20}$

Solution: factor top and bottom first, and then look to cancel common factors

$$\frac{12r^2-44r+40}{12r-20} = \frac{4(3r^2-11r+10)}{12r-20} = \frac{4(3r-5)(r-2)}{4(3r-5)} = r-2$$

Thus the answer is 14.671 – 2 or 12.671. No calculator needed!!!!

This ends the first lesson in working with rational expressions. Easy, eh?