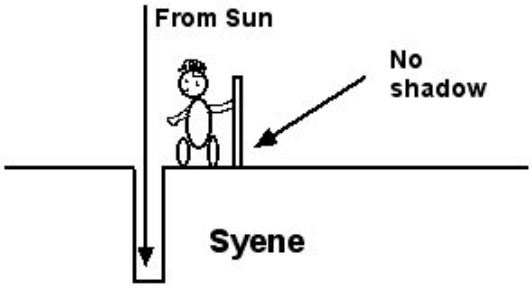
Eratosthenes Measures the Size of the Earth

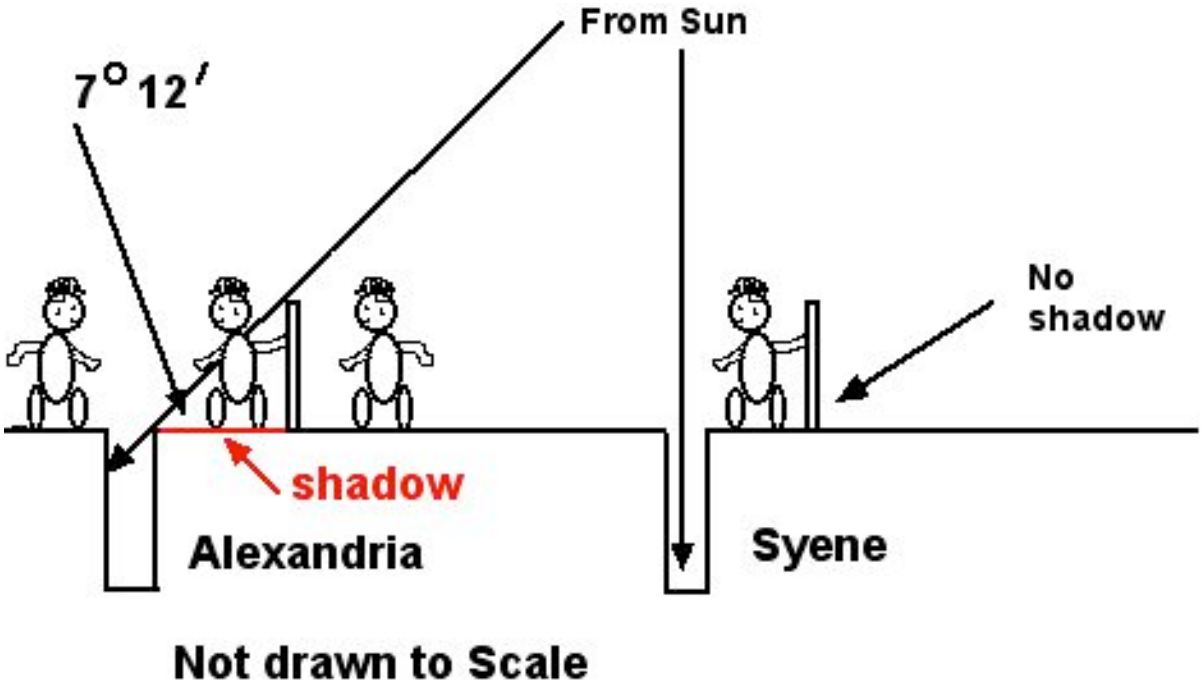
(Born Cyrene, about 276 BC, now Shahat, on the Libyan coast. Died, Alexandria, Egypt, about 196 BC)

This is another one of my Math "bedtime" stories. I really like it because it shows how some observation skills, coupled with grade 8 geometry, can bring you to wonderful conclusions.

About 240 BC (some 2 250 years ago), Eratosthenes went out to get a drink of water from a well. He noticed that he could see the bottom of the well, even though it was very deep. He also noticed that his staff, and he, did not have a shadow. Now this can only happen if the sun is directly overhead. Since he was in Syene (now Aswan) in Egypt, it was near the Tropic of Cancer, and at noon the sun is directly overhead. Eratosthenes thought that this was neat, and noted the date, and looked at his sand-dial wrist watch and noted the time. Here is a picture illustrating this happening.



The next year, when he was with some of his buddies on the same date, he said, "Hey, come outside and let's look down the village well, you can see the water down near the bottom. And you'll have no shadow!!". They couldn't believe him, so he dragged them outside to look down the well. Oops! The fact that he was in Alexandria, Egypt, 5 000 Greek stadia to the North, meant that it didn't work. Not only could they not see the water in the bottom of the well, they all had shadows! Eratosthenes measured the shadow of his staff, and using the new found trigonometric tables that he and his buddy Hipparchus had worked on, he found that the sun was at an angle of seven degrees, 12 minutes (7º, 12') away from the vertical. Now he really scratched his head, because he felt that he must be looking at the diagram below. And this would make the sun far too close, for he and Hipparchus had already worked out how far away the sun must be.



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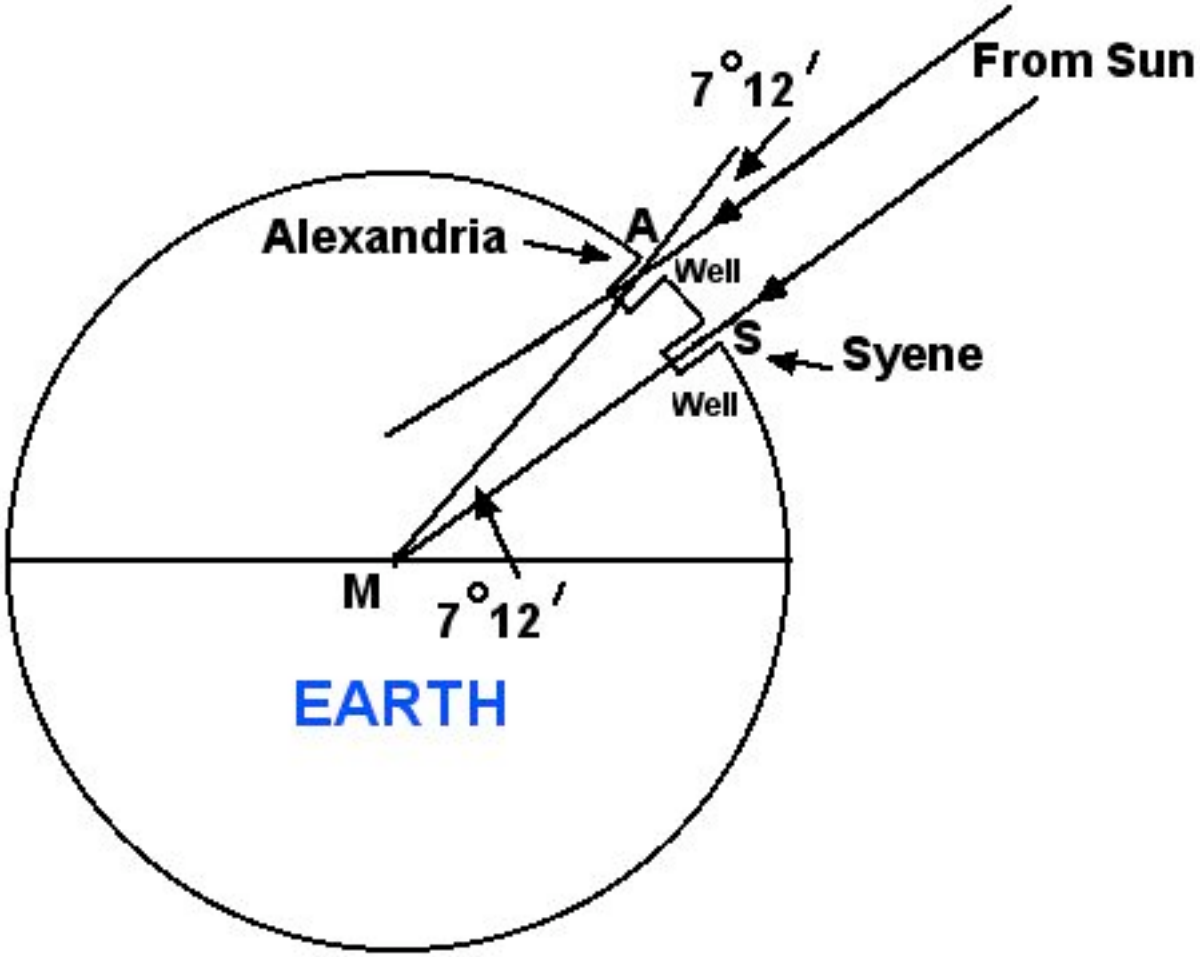
Suddenly, it dawned on him! This Earth thing that he was living on must not be flat, but round. And the diagram should be drawn as you see it below.



Because the sun is so far away, it's rays that hit the Earth are parallel. Also any deep well, dug straight down, is heading to the middle of the Earth (M). So the 7º 12'angle observed at Alexandria must, because of corresponding angles (grade 8 geometry), be the same at the centre of the Earth. So the angle at the centre of the earth from Syene (S), the Middle (M) and Alexandria (A), must also be 7º 12' or 7.2º. So, Eratostenes set up the following ratio: $\frac{7.2}{360} = \frac{5000\ \textbf{Stadia}}{\boldsymbol{C}}$. The ratio of 7.2º to the whole circle, 360º must be the same ratio as the distance from Syene to Alexandria (5 000 Greek Stadia) is to the Circumference (C) of the Earth. Now 7.2 divided by 360 equals one-fiftieth. So C = 50 times 5000, or 250 000 Greek Stadia, or 24 461 miles, or 39 186 km. The modern mean estimate, using satellites, computers and all sorts of modern instruments, is 24 891 miles or 39 875 km, about only 1.8% difference! Just by observing some shadows, taking a few measurements, and using a bit of geometry and logic. Wow!

Now, according to my sources, 1 Greek stadia = 600 feet or 182.88m. If this is the case, then the distance between the two wells would have been closer to 4 300 Greek Stadia. I'll continue to research this as I have found about three different lengths for a Greek stadia on the internet in only 10 minutes of trying!

Then why was Columbus so far off, 1492, from estimating the size of the earth?