Pythagorean Patterns in Carpentry

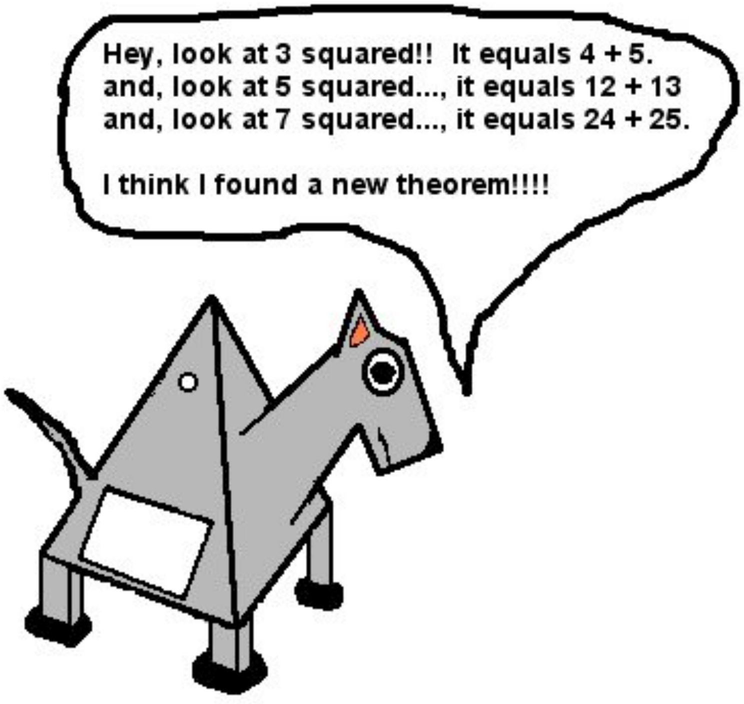
If you remember the Pythagorean Theorem states that, if ∆ABC is a right triangle, and “a” and “b” are the sides, and “c” is the hypotenuse, then: $a^2 + b^2 = c^2$

This leads to the simplest example of the 3 – 4 – 5 right triangle, since $3^2 + 4^2 = 5^2$, that is 9 + 16 = 25. If, the sides are all natural numbers, then we call this set 3 – 4 – 5 a Pythagorean Triple. Since, if I double or triple or multiply or divide all sides by the same thing, I get a similar triangle, then all multiples of 3 – 4 – 5 must also be a right triangle. In the table below, I have some examples of Pythagorean Triples and their multiples.

Pythagorean Triples and Their Multiples

Base Triple	3 – 4 – 5	5 – 12 – 13	7 – 24 – 25	8 – 15 – 17
Doubled	6 – 8 – 10	10 – 24 – 26	14 – 48 – 50	16 – 30 – 34
Tripled	9 – 12 – 15	15 – 36 – 39	21 – 72 – 75	24 – 45 – 51
Quadrupled	12 – 16 – 20	20 – 48 – 52	28 – 96 – 100	32 – 60 – 68
Halved	1.5 – 2 – 2.5	2.5 – 6 – 7.5	3.5 – 12 – 12.55	4 – 7.5 – 8.5
Multiplied by $\sqrt{2}$	$3\sqrt{2} - 4\sqrt{2} - 5\sqrt{2}$	$5\sqrt{2} - 12\sqrt{2} - 13\sqrt{2}$	$7\sqrt{2} - 24\sqrt{2} - 25\sqrt{2}$	$8\sqrt{2} - 15\sqrt{2} - 17\sqrt{2}$

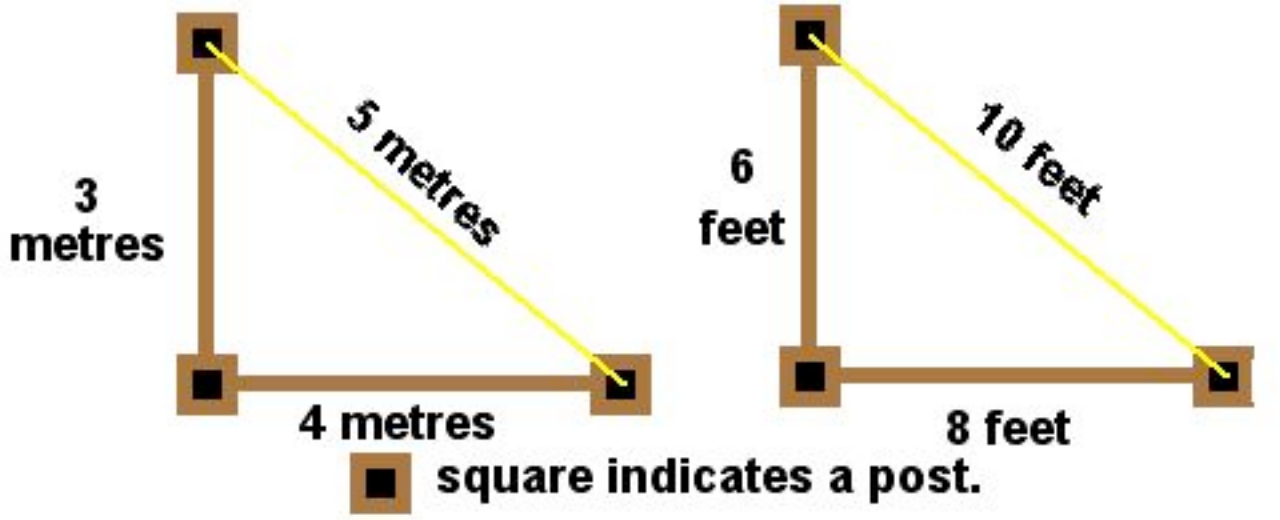
Now, for each of the triples the sum of the squares of the smaller two equal the square of the largest.



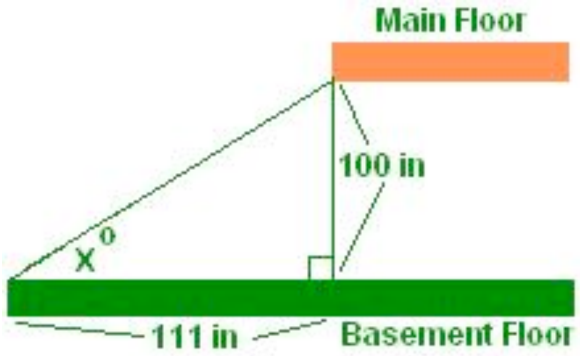
However, if you look closely, the square of the smaller side equals the sum of the other two sides! Well, almost. It works for the first three across the top, but not for 8 – 15 – 17, and the rest. But wait, the square of 8 is 64 and that is twice the sum of the other two sides! And the other two sides are two apart! Let’s try it on 9 – 12 – 15. The 12 and the 15 are 3 apart, and 9 squared equals 81 which is three times the sum of 12 and 15 (3 x 27 = 81). In fact it can be proved, (see proof below) that if “b” and “c” are “x” apart, then $a^2 = x(b + c)$. I think we should call this Tetra’s Theorem, however, I’m sure Euclid knew about it 2 300 years ago!

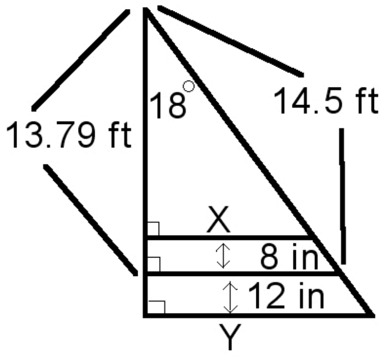
Here are some examples from the world of carpentry, that uses this Theorem.

When most buildings are started, you want to make sure you have a right angle for the corner. By looking at the two diagrams below, you can see why the carpenter is sure he or she has a right angle.

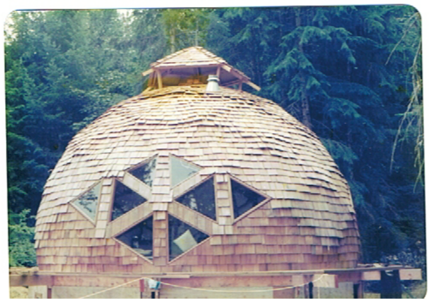


Since, both triangle are made up from Pythagorean triples, they will have a right angle in the lower left. In the diagram below left, I used Pythagoras when I wanted to build some stairs from the main floor to the basement floor. $100^2 + 111^2 = 22\ 321$, so the square root is 149.40 inches. Thus a 12 foot board is not long enough. We'll use this diagram later to work out the angle "x" using trigonometry.





On the right, we have another Pythagoras problem I had when building my footings. The length of the middle horizontal line had to be worked out with Pythagoras. I then used similar triangles (another lesson) to work out the length of my boards X and Y, to cut for my footings. When you design and build a weird house like I did, the geodesic dome pictured below, you can expect to use some math!!



Now for the proof of Tetra's Theorem:

Assume that we have a right triangle with small side equal to "a" and other two sides, "b" and "c" are "x" apart. That is, $c – b = x$. Transposing we get $c = x + b$.
Now, if it is a right triangle, then $a^2 + b^2 = c^2$. Substitute $x + b$ in for c and we get:

$a^2 + b^2 = (x + b)^2$, or squaring out the binomial, we have $a^2 + b^2 = x^2 + 2bx + b^2$

Now, if I subtract b^2 from both sides, I get : $a^2 = x^2 + 2bx$, or $a^2 = x^2 + 1bx + 1bx$

Rearranging the right hand side, I get: $a^2 = bx + x^2 + bx$.

Now finally factoring out the red x, I get : $a^2 = x(b + x + b)$. However, we know $x + b = c$, so

Substituting that in gives us : $a^2 = x(b + c)$, Q. E. D.

So Tetra's Theorem works!!