

Different Base Systems – Part Three

The basic idea of a place system for whole numbers is that the first column on the right is the unit's place, and the each column to the left has a value that is multiplied by the same thing each column.

Observe the table below.

n	Digits Used	n^6	n^5	n^4	n^3	n^2	n^1	n^0	Base 10 value
Base 10 decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9,	1 000 000	100 000	10 000	1 000	100	10	1	
		3	7	2	0	5	1	4	3 720 514
Base 8 octal	0, 1, 2, 3, 4, 5, 6, 7,	262 144	32 768	4 096	512	64	8	1	
				5	7	0	2	3	24 083
Base 2 binary	0, 1	64	32	16	8	4	2	1	
		1	1	0	1	0	1	1	107
Base 16 Hexa - decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E	16 777 216	1 048 576	65 536	4 096	256	16	1	
					1	C	5	8	7 256
Base 5	0, 1, 2, 3, 4	15 625	3 125	625	125	25	5	1	
				3	0	1	2	4	1 914

Some observations:

- (1) If you have base “n”, then there are “n” digits going from 0 to n – 1
- (2) The simplest base system is base 2 (binary), in that it only uses two digits, a “0” or a “1”.
- (3) Each column has “n” times the value of the column to its right.
- (4) The right hand column has a value of 1
- (5) To work out a Base 10 value of any number, follow this strategy:

Converting from any base to base 10 (decimal)

To convert from any base to a base 10 (decimal) system multiply each digit by the value of digit's “place”. Then add the results.

$$\begin{aligned}
 57\ 023_8 &= 5 \times 4\ 096 + 7 \times 512 + 0 \times 64 + 2 \times 8 + 3 \times 1 = \\
 &= 20\ 480 + 3\ 584 + 0 + 16 + 3 = 24\ 083
 \end{aligned}$$

Base 2 (binary) allowed all numbers to be written with two codes, a “0” or a “1”, a mark on a card, no mark on a card, a whole in a card, no whole in a card, and so on. It became the base system for computers, where each part of the memory was called a “byte” and it was made up from eight 0's and 1's, each called a bit (comes from **binary digit**). So a typical byte might look like this: 11010101 and a series of code would be: 11010101, 10001111, 01001101, 10000101, etc. This was hard to follow, so they took a byte and broke it up into two nibbles: 1101 0101 and using the following table:

nibble	0000	0001	0010	0011	0100	0101	0110	0111
Base 16	0	1	2	3	4	5	6	7
nibble	1000	1001	1010	1011	1100	1101	1110	1111
Base 16	8	9	A	B	C	D	E	F

The byte 1101 0101 in base 2 could be written as D5 in base 16 (Hexadecimal). This became a much more efficient way of writing out code, so Hexadecimal became an important base to work in, and, although we now use much higher levels of coding, you will see these numbers occurring quite often. An example is the codes for colours that you will see in a webpage source code.

Converting from a base 10 (decimal) to any other base

Convert 12 280 (base 10) to base 7 Put remainder here

Divide continually by the number that is the "base" you want to convert to.

Place answer below, and the remainder out to the right.

Keep dividing until this is "0"

→	$7 \overline{) 12\ 280}$	Rem
	$7 \overline{) 1\ 754}$	2
	$7 \overline{) 250}$	4
	$7 \overline{) 35}$	5
	$7 \overline{) 5}$	0
	$\overline{0}$	5

Read answer from bottom to top

Answer: 12 280₁₀ = 50 542₇

Put the base used here